

SKILL MISMATCH AND STRUCTURAL UNEMPLOYMENT*

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Abstract

I build a model in which structural change creates a mismatch between the skill requirements in the available jobs and workers' current skills. When the mismatch is severe, labor markets go through a prolonged adjustment process wherein job creation is low and unemployment is high. Due to matching frictions, firms find it harder to locate workers with the requisite skills for novel jobs and they respond by creating fewer jobs. The paucity of novel jobs increases unemployment for all workers—including those who already hold the requisite skills—and discourages skill acquisition by workers. Moreover, structural change interacts with the business cycle, causing a large and long-lasting increase in unemployment that concentrates in recessions. I demonstrate that the decline in routine-cognitive jobs outside manufacturing—a pervasive structural change that has affected U.S. labor markets since the late 90s—created a skill mismatch that contributed to the long-lasting increase in unemployment observed during the Great Recession. My evidence suggests that the amplification effects introduced by matching frictions are important. Moreover, I find that the skill mismatch has a larger effect during recessions and in labor markets where the demand for goods and services is depressed.

Keywords: Unemployment, Skill Mismatch, Structural Change, Great Recession.

JEL Classification: E24, E32, J21, J24.

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Structural change leads to the obsolescence of old jobs and expands novel jobs that embody new technologies. But the expansion of novel jobs does not guarantee that workers reallocate at no cost. Old and novel jobs require different skills, and a large share of the workforce lacks the skills that novel jobs require. Structural change can create a *skill mismatch*.

Since the mid 90s, cheaper computers have allowed firms to carry out tasks that previously had been performed by clerks, technicians, bookkeepers, salespersons, and other white-collar workers. This technological change led to a major restructuring of the labor market, characterized by a decline in routine-cognitive jobs that can be easily computerized because they follow precise procedures (see Autor, Dorn and Hanson, 2015). Figure 1 shows that, from 1996 to 2015, the share of the workforce employed in these office jobs declined from 25.5% to 21%—a 4.5 percentage points decline, or 7 million jobs. (The more publicized decline in manufacturing during the same period removed 9 million jobs.) The decline in routine-cognitive jobs coincided with the expansion of a wide range of professional jobs such as audio-visual specialists, executive secretaries, data administrators and analysts, computer support specialists and engineering jobs. Workers displaced from routine-cognitive jobs redeployed to professional jobs, but many of them lacked the analytical skills, training and formal education that are required in these jobs. By removing employment opportunities for middle-skill workers who specialized in routine tasks, the decline in routine-cognitive jobs created a skill mismatch.

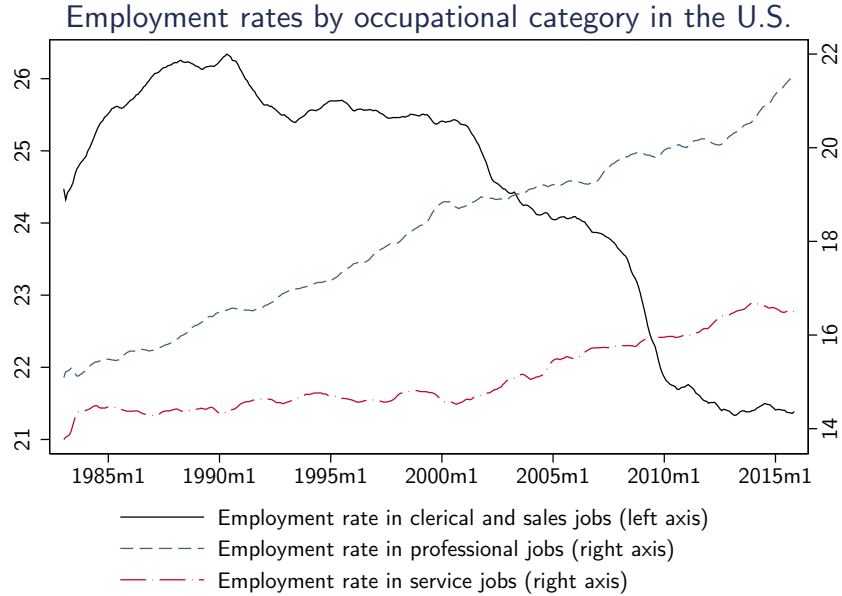


FIGURE 1: U.S. employment rates for different occupational categories. Data from the BLS.

In this paper I argue that when the skill mismatch is severe, labor markets go through a prolonged adjustment process in which job creation is low and unemployment is high for *all* workers. I also argue that the adjustment interacts with the business cycle, which causes a

long-lasting increase in unemployment that concentrates in recessions; booms, on the other hand, mask the negative consequences of structural changes. Using data for the U.S., I demonstrate that during the Great Recession, the decline in routine-cognitive jobs caused a skill mismatch that added to the large and long-lasting increase in unemployment.

In Section 1 I develop my argument. I study a model in which an economy adjusts to a one-time structural change that leads to the gradual obsolescence of old jobs—routine-cognitive jobs, in my example—and expands novel jobs that require different skills—professional jobs, in my example. The economy adjusts as the unskilled workers who lack the skills required in novel jobs retrain by taking stepping-stone jobs.

The key assumption in my argument is that labor markets are frictional and are characterized by bargaining over the product of labor. Firms and workers face matching frictions when they form jobs; once matched the worker and the firm bargain over the surplus of the relationship. Matching frictions are such that when firms post novel jobs they are less likely to obtain a skilled worker if the share of the unskilled among the unemployed is large. Thus, a surge in the share of unskilled workers that are searching for novel jobs—a severe skill mismatch—reduces the ability of firms to locate workers with the requisite skills for novel jobs and crowds out matching opportunities for skilled workers. I think about this assumption in terms of the following metaphor: firms may be able to target their search efforts based on imperfect signals—a test, an interview, a resume or referral, a job requirement—because firms either only observe these signals or their search technology cannot be perfectly directed. Conditional on a given signal, the employer’s posterior probability that a worker is skilled decreases when the unskilled are numerous (see Coate and Loury, 1993; Acemoglu 1996), which reduces the ability of the firm to locate skilled workers.

Structural change causes temporary but long-lasting unemployment. Because they lack the skills required in novel jobs, unskilled workers go through a prolonged period of unemployment and low wages until they retrain. I refer to this as the *direct effect* of structural change, which is the usual mechanism emphasized in the literature on job displacement and reallocation (see Kambourov and Manouskii, 2009; Alvarez and Shimer, 2011; and Jaimovic and Siu, 2014).

The direct effect is only a part of the story. Matching frictions introduce two amplification mechanisms that depend on the aggregate extent of skill mismatch—the share of unskilled workers among those who are searching for novel jobs. There is a *job creation externality* that amplifies unemployment. The skill mismatch lowers employer’s expectations about the probability of obtaining a skilled worker, and firms respond by creating less novel jobs. Thus, when the unskilled abound, the skill mismatch lowers the job-finding rate of both skilled and unskilled workers. This constitutes an externality because changes in the finding rate of workers have a first-order effect on their utility. Moreover, there is a *complementarity effect*, which dampens retraining and prolongs the adjustment. Since retraining is only useful in

novel jobs, during periods of skill mismatch, the paucity of novel jobs reduces the incentives of workers to retrain. Firms respond by creating few stepping-stone jobs even if there are no contractual frictions or nominal rigidities involved (see Caballero and Hammour, 1996).

The distinctive implication of my model is that, due to the job creation externality and the complementarity effect, the finding rate of a worker not only depends on his skills, but also on the skill mismatch in his labor market. The skill mismatch reduces job opportunities for skilled workers and affects the redeployment and retraining of unskilled workers

Although the decline in routine-cognitive jobs started in the mid 90s, Figure 1 shows that about two thirds of the decline in the last 20 years occurred during the Great Recession (see also the evidence by Jaimovic and Siu, 2014). Also, as I show in my empirical exercise, the effects of this structural change on unemployment concentrate in economic downturns. My framework underscores two potential mechanisms by which the structural change interacts with the business cycle, causing a long-lasting increase in unemployment that concentrates in recessions.

First, because unskilled workers produce a low surplus in the available jobs, their job-finding rate is more responsive to changes in productivity.¹ Thus, during periods of low aggregate productivity, unskilled workers cannot find novel jobs easily; the share of unskilled workers among the unemployed rises, which exacerbates the skill mismatch and its negative externalities (see also Pries, 2008, who emphasizes the same mechanism).

Second, a literature going back to Schumpeter (1942) argues that, due to the low opportunity cost of adjustment during recessions, firms use crisis to replace old jobs with new technologies or restructure and close job positions that will soon become obsolete due to advances in technology.² In the case of routine-cognitive jobs, the data supports the assumption that firms adjust their labor requirements during recessions, which caused a permanent shift in the demand for routine-cognitive labor during the Great Recession. Figure 2 shows that during the recession, job openings for routine-cognitive jobs—the analog of old jobs in my model—suffered a permanent decline of about 55% relative to other jobs.³

I incorporate this possibility by assuming that recessions bring a temporary increase in the rate at which firms permanently close the available positions for old jobs and stop hiring labor to produce these tasks. During good times, old jobs are still plentiful; workers are not displaced nor forced to redeploy to novel jobs and the skill mismatch is modest. During

¹Hagedorn and Manouski (2008) and Ljungqvist and Sargent (2015), too, emphasize that when the net surplus is low, wages become endogenously rigid and the job-finding rate of workers becomes more cyclical.

²See also Davis and Haltinwanger (1990), Hall (1991), Caballero and Hammour (1994), Aghion and Saint Paul (1998), Koenders and Rogerson (2005), and Berger (2014).

³A recent report by Burning Glass Technologies (2014) shows that, even within the remaining job openings for middle-skill jobs, firms are demanding higher qualifications and workers are expected to perform different tasks than before.

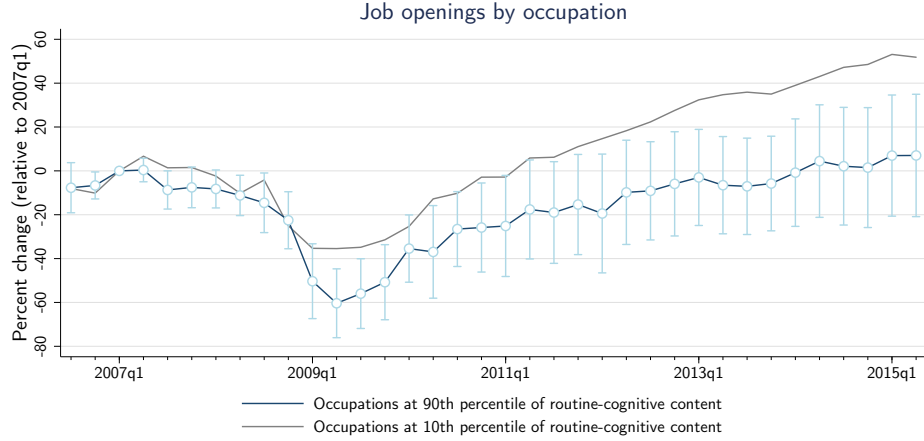


FIGURE 2: Percent change in quarterly vacancies posted within broad occupational groups (relative to the first quarter of 2007). The light-blue bars plot 90% confidence intervals for the difference between both series in each quarter. Data for 22 occupations from Help Wanted Online, by the Conference Board.

recessions, firms adjust the type of jobs that they post and there is a permanent decline in openings for old jobs. The paucity of old jobs pushes unskilled workers into unemployment and to redeploy to novel jobs, which exacerbates the skill mismatch and its negative externalities. I show that when training costs are high, the resulting increase in unemployment may outlast the recession and create a jobless recovery.

The theory section presents the formal statements and intuitions for these results. The model is dynamic and has several state variables, but a careful choice of assumptions allows me to characterize the adjustment path when gross flows are sufficiently large (as is the case in the data for the U.S.). Moreover, I characterize the inefficiencies that arise because of the job creation externality in terms of wedges between the private and social value of job searching for different workers (as in Shimer and Smith, 2001). The wedges indicate that the inefficiencies arise because unskilled workers deteriorate matching opportunities for all workers, while skilled workers improve them. Workers and firms do not internalize these first-order effects when forming matches or creating stepping-stone jobs. The inefficiencies arise even if the Hosios condition holds (Hosios, 1990).

Section 2 supplements my theoretical analysis with a quantitative exploration of the model. For plausible parameters, the external and complementarity effects explain about 40% of the increase in unemployment along the adjustment. The efficient allocation exhibits about 30% less unemployment than the decentralized one and features more stepping-stone jobs. Although firms and workers bargain over small quasi-rents (matching frictions are small in the parametrization I use), matching frictions can have a significant effect on skill acquisition and job creation. Moreover, the interaction between structural change and recessions is significant. I parametrize a recession as a decline of 5% in productivity and an increase in the rate at which

firms close available positions for old jobs. This shock is calibrated to match the permanent decline in job openings for routine-cognitive jobs that I present below in Figure 2. Both shocks last for 10 quarters. When the recession affects an economy that is adjusting to structural change, it increases unemployment by up to 3 percentage points. Five years after the recession ends, unemployment remains above its pre-recession trend for both types of workers. In the absence of structural change, the same recession would only increase unemployment by 1 percentage point, and unemployment would exhibit no propagation (see Shimer, 2005).

Section 3 presents my empirical analysis of the decline in routine-cognitive jobs in the United States. In line with the patterns in Figures 1 and 2, I find that the decline in routine-cognitive employment concentrated during the Great Recession and it prompted the redeployment of workers who specialized in these jobs to professional jobs. This structural change brought a skill mismatch which contributed to the large and long-lasting increase in joblessness that was observed in the U.S. during the Great Recession and its recovery (from 2007 to 2013).

The key empirical implication of my framework is that, due to the job creation externality, the finding rate of a worker not only depends on his skills—the direct effect—, but also on the extent of the skill mismatch in his labor market, which reduces the creation of novel and stepping-stone jobs and amplifies unemployment. My empirical strategy separates the direct effect of the skill mismatch from the job creation externality and the complementarity effect—the external effects of skill mismatch for short—that operate in a local labor market.

To estimate the direct effect of structural change, I partition workers in the American Community Survey into 200 skill groups (defined by age, education, sex and region of residence) and measure their specialization in routine-cognitive jobs using their year 2000 share of employment in these jobs.⁴ As expected of the direct effect, from 2007 to 2013, workers in skill groups at the 90th percentile of specialization in routine-cognitive jobs—the analog of unskilled workers in my model—suffered a decline of 2.4 percentage points in their employment rate relative to workers in skill groups at the 10th percentile.

To estimate the external effects of the skill mismatch, I compare workers in the same skill group who reside in commuting zones—or local labor markets—exposed to different levels of skill mismatch.⁵ I measure the skill mismatch in each commuting zone by the share of workers who in 2000 were employed in routine-cognitive jobs—the analog of the share of

⁴This builds on work by Acemoglu and Autor (2011) and Foote and Ryan (2014), who follow the same procedure to study the effects of technology on workers in different skill groups.

⁵This approach builds on the work of Autor, Dorn and Hanson (2013, 2015), who explore the aggregate consequences of trade and technology on local labor markets. Moreover, a growing literature documents that the impact of shocks that require reallocation of workers are more visible not in national-level outcomes, but in the most exposed commuting zones (see Acemoglu et al. 2014; and Autor, Dorn and Hanson, 2015). These authors argue, as I do here, that this reflects adjustment costs that affect all the workers in a labor market.

unskilled workers in my model. As expected of the external effects, I find that workers who were located in commuting zones that had a worst skill mismatch suffered from a more long-lasting decrease in employment than workers *in the same* skill group who were located in other zones. These external effects are significant and may explain about half of the decrease in employment associated with the decline in routine-cognitive jobs. I complement this evidence with an instrumental-variable strategy and a placebo test using the 1990 recession, which show that the estimated external effects do not capture unobserved differences across commuting zones.

My findings imply that, during the Great Recession, unemployed workers in the most exposed commuting zones experienced a large and long-lasting decline in their job-finding rate. Figure 3, which uses data from the Longitudinal Employer-Household Dynamics, previews this result. Workers in commuting zones at the 90th percentile of mismatch suffered a persistent 15% additional decline in their job-finding rate from 2007 to 2014, relative to workers in zones at the 10th percentile of mismatch. This represents an additional 1.3 percentage points decline in their employment rate by 2014.

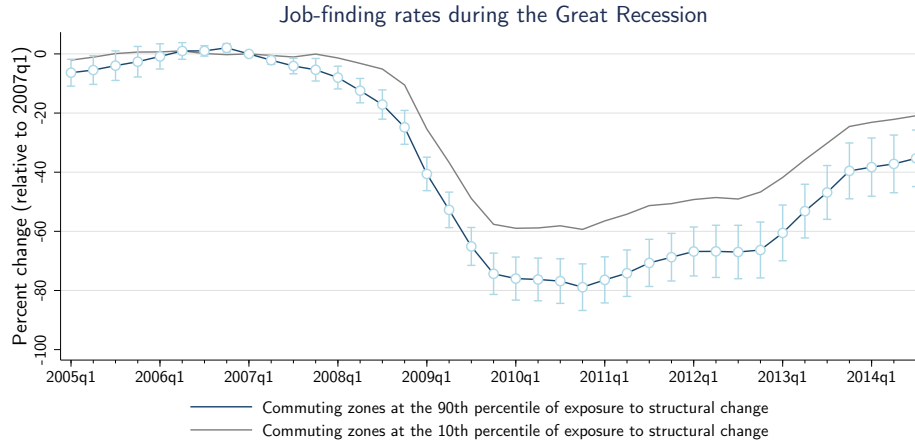


FIGURE 3: Percent change in unemployed workers’ job-finding rate (relative to the first quarter of 2007). The light-blue bars plot 90% confidence intervals for the difference between both series in each quarter. Data from the Longitudinal Employer-Household Dynamics.

The incidence of the external effects provides more concrete evidence of the external effects of the skill mismatch. The job creation externality and the complementarity effect should affect workers who specialize in professional jobs and workers that need to redeploy to these jobs. When I estimate the incidence of the external effects of skill mismatch, I find that they are associated with a reduction in employment for groups of workers who specialized in professional jobs—the empirical analog of skilled workers in my model—and a reduction in employment and wages of workers who specialized in routine jobs. For workers displaced from routine-cognitive jobs, the external effects manifest partly as a lower probability of successfully

reallocating to professional jobs, which is consistent with the paucity of stepping-stone jobs predicted by my model. Remarkably, I do not find evidence of an external effect of skill mismatch on workers specialized in service or managerial jobs.

To support my interpretation of the evidence, I evaluate alternative explanations for the existence of amplification effects at the commuting-zone level. Hypothetically my estimates could capture local demand externalities (see Beaudry, Galizia and Portier, 2014), or they could result from the decreasing marginal value of jobs receiving displaced workers. But none of these alternatives explain my findings. The estimated negative external effects are present among workers who specialize in the tradable sector and are not present on workers who specialize in the non-tradable service jobs, which rules out an explanation based on local demand externalities. In addition, the estimated negative external effects are robust when I control for changes in employment by occupation, which deals with potential changes in task prices that stem from decreasing returns.

Finally, my evidence suggests that the secular decline in routine-cognitive jobs interacted with the Great Recession. Although the effects of the skill mismatch were small or negligible before the Great Recession, during the recession and its recovery the effects are large and significant, as anticipated in Figure 3. The same occurs during the 2001 recession, but not during the 1990 recession, which preceded the decline in routine-cognitive jobs. Moreover, I find that reductions in local economic activity—proxied by a decline in household net worth in a commuting zone (see Mian and Sufi, 2014)—interacted with the skill mismatch. In the commuting zones most exposed to structural change, the decline in local demand exacerbated the skill mismatch by accelerating the decline in routine-cognitive jobs. Through this interaction, the decline in local demand had a large and persistent effect on employment that lasted up to 2013. In the least exposed commuting zones, the decline in local demand had a modest and short-lived impact on employment, which fully vanished by 2013.

Related literature: The mechanism behind the job creation externality builds on the work of Acemoglu (1996, 1997), who presents models in which the ex-post bargaining of workers and firms over their joint surplus reduces investments in capital and training.⁶ My paper incorporates this mechanism into the canonical search model, which allows me to quantify the externalities that arise solely from the small matching frictions that are typically calibrated in the literature. Unlike previous studies, I examine if this mechanism affects how the economy adjusts to structural changes and if this mechanism interacts with the business cycle. I also

⁶In a related paper, Coates and Loury (1993) argue that imperfect learning by employers creates a negative spillover on all workers and reduces incentives for skill acquisition. Shimer and Smith (2001) also emphasize the role of externalities in a matching model with ex-ante heterogeneous agents. Beaudry, Green and Sand (2012) provide evidence that openings for high-paying jobs create a positive externality, as the job creation externality does in my model.

provide evidence of the external amplification effects implied by this mechanism.

My paper contributes to the literature on job polarization (Autor, Levy and Murnane, 2003; Goos and Manning, 2007). In a recent paper, Autor, Dorn and Hanson (2015) explore the aggregate consequences of polarization on employment from 1990 to 2007, when the decline in routine-cognitive jobs did not have a major impact. Closest to my paper is a study by Jaimovic and Siu (2014), who argue that employment polarization interacted with the last three recessions and generated jobless recoveries. Their findings and mine are complementary, but our studies differ in several respects. Jaimovic and Siu present evidence that is based on employment counts at the national level, and they focus on the decline of all middle-skill jobs, which includes manufacturing jobs. Their findings were criticized by Foote and Ryan (2014) on the grounds that their time series patterns could be explained by differences in cyclicity among manufacturing industries. By focusing on the reduction of routine-cognitive jobs outside manufacturing and controlling for differences in industry cyclicity, my empirical approach overcomes this criticism. Unlike my model, in the Jaimovic and Siu (2014) model there are no externalities that amplify unemployment, which is driven solely by the assumption that displaced workers have a low matching efficiency. Also different in the two models is the nature of the interaction with recessions. In the Jaimovic and Siu (2014) model, recessions increase the separation rate for middle-skill workers, and, in contrast to my model, once productivity recovers the finding rate of workers returns to its trend—there is no propagation.

My paper also contributes to the literature that examines how sectoral or occupational shocks, as opposed to aggregate shocks, drive unemployment fluctuations.⁷ This literature goes back to Lillien (1982) and re-emerged with the debate over whether unemployment during the Great Recession reflected a sectoral or occupational mismatch between available jobs and unemployed workers (see Kocherlakota, 2010). Using U.S. data, Chodorow-Reich and Wieland (2015) construct a measure of sectoral reallocation at the local labor market and show that this reallocation contributes to worst employment outcomes, especially during recessions (see also Garin, Pries and Sims, 2013; and Mehrota and Sergeyev, 2013). Using a decomposition based on aggregate data, Sahin et al. (2014) find a smaller role for industry mismatch or sectoral shocks in explaining unemployment during the Great Recession, though they find some role for occupational mismatch. However, the role of occupational or skill mismatches remains a matter of debate (see Lazear and Spletzer, 2012; and Wiczer, 2013).

I contribute to this literature by showing that the lack of skills among displaced routine-cognitive workers—a source of occupational or skill mismatch—contributed to the large and

⁷The literature on unemployment that arises from reallocation started with Lucas and Prescott (1976). Recent papers focus on the stationary properties of unemployment in models of reallocation and technological change (see Aghion and Howitt, 1994; Mortensen and Pissarides, 1998; Alvarez and Shimer, 2009; and Birchenall, 2011). Carrillo-Tudela and Vischers (2014) also study reallocation during the business cycle.

prolonged increase in unemployment during the Great Recession.⁸ The external effects of the skill mismatch mean that the impacts of the decline in routine-cognitive jobs are most visible not in national-level outcomes, which are the focus of the existing literature on mismatch, but in the commuting zones that were more exposed to this structural change, which are the focus of this paper.

Finally, I contribute to the literature that examines the empirical performance of Mortensen and Pissarides’s (1994) matching model. I show that through their interaction with structural change, recessions can generate a large and long-lasting increase in unemployment. As shown by Shimer (2005) the canonical search model by itself fails to generate these patterns.⁹ In keeping with the available evidence, due to their lack of requisite skills, unemployment spells for unskilled workers who have been displaced from old jobs may be more costly than in the canonical search model (see Davis and von Wachten, 2011).¹⁰

1 A MODEL OF STRUCTURAL CHANGE

I extend the matching model of Mortensen and Pissarides (1994) to include several types of jobs and workers. Time is continuous and I omit it whenever it causes no confusion. All individuals are risk neutral and discount the future at a constant rate r .

In its status quo, the economy produces a final good Y (with price normalized to 1) by combining a mass of tasks $y(i)$ with $i \in [0, 1]$:

$$Y = \int_0^1 y(i) di.$$

A mass 1 of workers are employed in jobs, each of which produces a single task i .

Structural change shifts the productive structure by an amount Δ as shown in Figure 4.

The shift in the productive structure partitions the task space in a variety of jobs:

Old jobs—indexed by the superscript o —, which produce tasks in $[0, \Delta)$. These jobs are at risk of becoming obsolete due to the competition from technology or because these jobs

⁸Kroft et al. (2014) and Barnichon and Figura (2015) emphasize the role of pure duration dependence as opposed to worker ex-ante heterogeneity or amplification effects of the sort that I propose. However, these approaches assume there is no unobserved heterogeneity or externalities that affect job creation.

⁹Shimer’s paper sparked a whole literature that modified the canonical search model or the calibration used to improve the model’s ability to match the data (see Hall, 2005; Hagedorn and Manouski, 2008; Hall and Milgrom, 2008; Pissarides, 2009; and Ljunquist and Sargent, 2015).

¹⁰In my model, unemployment spells for unskilled workers are more costly during recessions and when markets face a severe skill mismatch. The reason is that the paucity of novel jobs affects the redeployment of unskilled workers. This provides an alternative perspective to recent scholarship (see Huckfeldt, 2014; Jarosch, 2014; and Krolikowski, 2014), which emphasizes how recurrent job losses create costly unemployment spells.

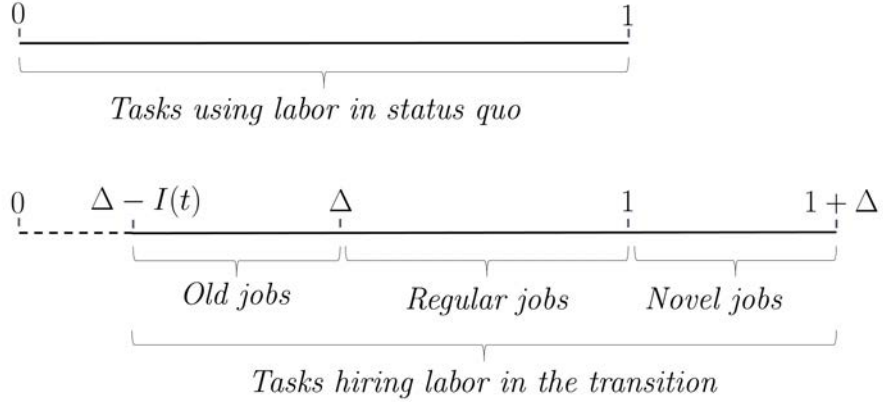


FIGURE 4: Task space and a graphical representation of the effect of structural change on the productive structure. The top panel presents the status quo and the bottom panel the structural change.

embody old technologies. In the example in the Introduction, old jobs correspond to the routine-cognitive jobs.

Old jobs in $[\Delta - I(t), \Delta]$ still hire labor. $I(t)$ determines the number of available old jobs. These jobs become obsolete at an exogenous rate $v(t) > 0$, so that

$$\dot{I} = -Iv(t), \text{ with } I(0) = \Delta.$$

Old jobs in $[0, \Delta - I(t))$ are obsolete and do not hire labor.

The rate $v(t)$ may be constant during normal times, which reflects the secular advancement of structural change. Increases in $v(t)$ reflect fast technological change or periods of adjustment in which firms adopt the existing technologies and permanently close positions for old jobs, such as during recessions.

Regular jobs—indexed by the superscript r —, which perform tasks in $[\Delta, 1]$. Structural change does not affect these jobs. Regular jobs do not require new skills, and they provide feasible employment alternatives for workers displaced from old jobs. In the example cited in the Introduction, regular jobs correspond to service jobs that do not require retraining.

Novel jobs—indexed by the superscript n —, which perform tasks in $(1, 1 + \Delta]$. Structural change expands these additional jobs. The defining characteristic of novel jobs is that they require additional skills which unskilled workers lack. In the example described in the Introduction, novel jobs correspond to professional jobs that rely on analytical and cognitive skills, and tend to require more training, job-related experience and formal education than routine-cognitive jobs.

Among novel jobs we have *stepping-stone jobs*, which provide retraining opportunities that allow unskilled workers to become skilled on the job. I think of stepping-stone jobs as created

by firms that have enough time and resources to undertake costly investments to train new hires. For instance, a stepping-stone job for secretaries would allow them to learn over time the skills needed to become an executive secretary. Or firms could train technicians to perform some of the tasks reserved for professional engineers. Stepping-stone jobs become an attractive option for firms when unskilled workers abound and skilled workers are hard to find. Though the firm must incur training costs and wait for workers to become skilled, it can extract in the form of lower wages part of the gains of training, especially when unskilled workers highly value the acquisition of new skills (see Becker, 1964).

On the labor supply side, workers are of two types: skilled—indexed by the subscript s —or unskilled—indexed by the subscript u . Skilled workers produce $z(t)$ units of any task. Unskilled workers produce $z(t) - q^n$ units of new tasks and $z(t)$ units when employed in regular or old jobs. Here, $z(t)$ is the marginal product of labor and $q^n > 0$ reflects unskilled workers' lack of expertise in novel jobs (it is in this sense that workers are unskilled). By taking stepping-stone jobs unskilled workers retrain and become skilled through on-the-job learning. When employed in these jobs, unskilled workers produce $z(t) - q^n - q^l$ units of output and become skilled at a rate $\alpha > 0$. Here, $q^l > 0$ denotes the costs of training. Moreover, unskilled workers become skilled at a small exogenous rate $\delta \approx 0$, which captures other forces not modeled that can include the entry of new college cohorts or the standardization of new technologies. This exogenous rate guarantees that all the workforce eventually becomes skilled.

As old jobs obsolesce and close, firms stop hiring labor for these jobs and workers redeploy to novel jobs. The skill mismatch arises because unskilled workers are less productive at novel jobs. The economy adjusts as unskilled workers take stepping-stone jobs and become skilled, so that in the long run, all workers are employed in jobs that produce tasks in $[\Delta, 1 + \Delta]$.

To introduce my mechanism, I depart from competitive labor markets and assume that there are matching frictions. For each task, there is a separate *hiring market*. Unemployed workers populate these hiring markets in an undirected fashion, constantly churning across markets, tinkering at job opportunities in different tasks until they are matched to a job opening. When unemployed, skilled workers spend a share Δ of their time searching in the hiring markets for novel jobs, and they spend the remaining share $1 - \Delta$ searching in hiring markets for regular jobs. These frequencies reflect the share of regular and new tasks in the economy. Unskilled workers also take advantage of available old jobs. They spend a share $\frac{\Delta}{1+I}$ of their time searching in the hiring markets for novel jobs; a share $\frac{1-\Delta}{1+I}$ in the markets for regular jobs; and the remaining share $\frac{I}{1+I}$ searching in the hiring markets for the available old jobs. Thus, at each point in time, the hiring market for old jobs is populated by unskilled workers, while the hiring markets for regular and novel jobs are populated by both types of workers.

To hire workers, firms post vacancies at a flow cost κ , which are aimed at a particular type

of worker. When hiring for novel jobs in $i \in [1, 1 + \Delta]$, firms may post stepping-stone jobs aimed at unskilled workers, $v_u^l(i)$. As emphasized above, the creation of stepping-stone jobs is the main endogenous margin that drives the adjustment of the economy.

Alternatively, firms can attempt to hire workers who already hold the requisite skills by posting vacancies for jobs that do not offer any training, $v_s^n(i)$, and that are aimed at skilled workers. The key assumption is that, when posting these jobs, firms cannot perfectly direct their search efforts, and so with some random probability they will be (mis)matched to an unskilled worker who is searching in the hiring market for task $i \in [1, 1 + \Delta]$. To model this possibility, I assume that with probability $\pi > 0$, unskilled workers fail to be screened out and end up in the queue for vacancies $v_s^n(i)$. This allows some unskilled workers to obtain jobs faster, although these jobs do not offer training. In this case, firms only realize they were mismatched after meeting with the worker and having already incurred in search costs. With probability $1 - \pi < 1$, unskilled workers reveal their true type and queue only for stepping-stone jobs. Skilled workers who search for novel jobs always queue for the vacancies $v_s^n(i)$. Thus, firms that post vacancies $v_s^n(i)$ are randomly matched to both skilled and unskilled workers at frequencies that depend on π and on the number of skilled and unskilled workers who are searching for novel jobs.

My model captures succinctly how random matching affects the probability with which firms expect to obtain a skilled match when posting a novel job. Let γ^n denote this probability, and γ be the share of skilled workers among the unemployed. Then

$$\gamma^n = \frac{\gamma \Delta}{(1 - \gamma) \frac{\Delta}{1+I} \pi + \gamma \Delta}. \quad (1)$$

When the unskilled abound among the unemployed (γ is low) and structural change makes more old jobs obsolete (I declines and pushes unskilled workers to redeploy to novel jobs), firms become pessimistic about obtaining a skilled match and about the expected profits from job creation, captured here by a reduction in γ^n . Lower values of γ^n reflect a severe skill mismatch.

The defining feature of random matching is not that firms cannot direct their search efforts (they could wait for a skilled match if they wanted to), but that an inflow of unskilled workers—the skill mismatch—crowds out matching opportunities for skilled workers and increases the risk for firms of being mismatched (see Shimer and Smith, 2001). Equation (1) captures this feature succinctly in a reduced form way. Here, π represents the noise in the signals used by firms to screen candidates or the extent to which firms cannot perfectly direct their search efforts. Both of which reduce firms' ability to locate skilled workers during periods of severe mismatch.¹¹

¹¹To think about the role of the random matching assumption, I find it useful to consider the case of a *young*

For tractability, and because they play no major role, the interactions in the hiring markets for regular and old jobs are simpler. In the market for regular jobs, firms post vacancies that are aimed at hiring skilled workers, $v_s^r(i)$, or unskilled ones $v_u^r(i)$, for all $i \in [\Delta, 1)$. Firms are able to separate workers by their type, and skilled workers in this market queue for vacancies $v_s^r(i)$; while unskilled workers queue for vacancies $v_u^r(i)$. Finally, in the hiring market for old jobs, firms post vacancies aimed at the unskilled workers that populate it, $v_u^o(i) \forall i \in [\Delta - I(t), \Delta)$. Hiring does not feature a random component in these markets.

When firms post a vacancy $v_j^k(i)$, they are matched to the workers in the queue for the job at a rate $q(\theta) = a\theta^{-\eta}$, with $\eta \in [0, 1]$. Here, θ —the *tightness*—equals the ratio of vacancies to the number of workers who are searching for this particular job. Workers queuing for a vacancy are matched at a rate $f(\theta) = a\theta^{1-\eta}$. Thus, the matching process in each hiring market exhibits constant returns to scale. Once matched, the firm observes the worker type and decides whether to keep the match. If they do so, they split the surplus through Nash bargaining and the worker obtains a share $\beta > 0$. Ongoing matches separate at an exogenous rate $\lambda > 0$ and there are no endogenous separations. Finally, there is free entry of firms.

To complete the description of the environment, I now present the behavior of the state variables. I have to keep track of $x(t)$, the number of skilled workers; $u(t)$, the unemployment rate; $s(t)$, the number of stepping-stone jobs; $\gamma(t)$, the share of skilled workers among the unemployed; and $I(t)$, the remaining old jobs. The state variables evolve according to the backward-looking differential equations:

$$\begin{aligned} \dot{x} &= \alpha s + \delta u(1 - \gamma), & \dot{u} &= \lambda(1 - u) - u\gamma f_s - u(1 - \gamma)f_u, \\ \dot{s} &= u(1 - \gamma)\frac{\Delta}{1 + I}(1 - \pi)f(\theta_u^l) - (\alpha + \lambda)s, & \dot{\gamma} &= (1 - \gamma)\gamma(f_u - f_s) + \lambda\frac{x - \gamma}{u} + (1 - \gamma)\delta, \\ \dot{I} &= -Iv(t). \end{aligned} \tag{2}$$

Here, θ_u^l is the tightness in the queue for stepping-stone jobs. f_u and f_s are unskilled and skilled workers finding rates, respectively, which depend on the equilibrium tightness in all labor markets. The finding rates are given by (assuming all matches yield a positive surplus):

$$\begin{aligned} f_s &= \Delta f(\theta_s^n) + (1 - \Delta)f(\theta_s^r), \\ f_u &= \frac{\Delta}{1 + I}[(1 - \pi)f(\theta_u^l) + \pi f(\theta_s^n)] + \frac{1 - \Delta}{1 + I}f(\theta_u^r) + \frac{I}{1 + I}f(\theta_u^o). \end{aligned} \tag{3}$$

firm that has just entered the market and is deciding whether to create a professional job. This firm has no large human-resources department, it does not receive hundreds of job applications from the best workers, and cannot go through long and costly processes to select its personnel. Such a firm must take chances, and will choose to expand depending on the expected skill level of workers who are searching for jobs. This example is relevant because the evidence suggests that *young* and *new* firms are responsible for the bulk of employment growth (see Haltinwanger, Jarmin and Miranda, 2011).

Given a starting value for the state variables $\{x(0), u(0), s(0), \gamma(0), I(0)\}$, an *allocation* consists of a path for tightness $\{\theta_s^n(t), \theta_u^l(t), \theta_s^r(t), \theta_u^r(t), \theta_u^o(t)\}$, and a path for the state variables $\{x(t), s(t), u(t), s(t), I(t)\}$ that solves the system of differential equations given by their initial condition and equation (2).

1.1 Characterizing the Equilibrium

An equilibrium is given by an allocation in which the tightness of all markets is determined by firm entry decisions, and firms enter the market motivated by the profits from job creation.

The surplus of different matches S_j^k — k indexes the type of job and j the type of worker—satisfy the Bellman equations:

$$\begin{aligned} (r + \lambda)S_s^n &= z(t) - (rU_s - \dot{U}_s) + \dot{S}_s^n, & (r + \lambda)S_s^r &= z(t) - (rU_s - \dot{U}_s) + \dot{S}_s^r, \\ (r + \lambda)S_u^n &= z(t) - q^n - (rU_u - \dot{U}_u) + \dot{S}_u^n, & (r + \lambda)S_u^r &= z(t) - (rU_u - \dot{U}_u) + \dot{S}_u^r, \\ (r + \lambda)S_u^o &= z(t) - (rU_u - \dot{U}_u) + \dot{S}_u^o, & & \end{aligned} \quad (4)$$

The discounted surplus on the left equals the flow value of production, minus the opportunity cost of workers (their reservation wage, $rU_j - \dot{U}_j$), plus the appreciation of the match value. Free entry by firms implies that their opportunity cost of engaging in a match is zero.

The surplus of stepping-stone jobs has a different Bellman equation that is given by:

$$(r + \lambda)S_u^l = z(t) - q^n + \max\{-q^l + \alpha(U_s - U_u) + \alpha(S_s^n - S_u^l), 0\} - (rU_u - \dot{U}_u) + \dot{S}_u^l. \quad (5)$$

The terms $\alpha(S_s^n - S_u^l)$ and $\alpha(U_s - U_u)$ correspond to the gains, shared by the worker and the firm, when the worker becomes skilled. The max operator on the right-hand side indicates that firms have the option value of not training workers if it is not profitable for the pair. The term $\alpha(U_s - U_u)$ underscores the fact that workers recognize the benefits of taking these jobs and they share these benefits with the employer through bargaining, who is then able to recover part of the training expenditures. No contractual problems affect stepping-stone jobs.¹² However, as it will be clear in my analysis, when forming these jobs the firm and worker do not take into account the benefits that accrue to future employers, who benefit from the better chances of matching with a skilled worker.

¹²One could incorporate these inefficiencies by assuming that with some probability $H > 0$, the worker captures the value of the increase in his outside option. This could also represent a lower bound on wages. I find that small values of H reduce training, exacerbate the skill mismatch, and have a large effect on unemployment. See also Caballero and Hammour (1996) for models in which contractual problems slow down the adjustment of the economy.

Workers' reservation wages are given by the Bellman equations:

$$\begin{aligned} w_s &= rU_s - \dot{U}_s = b + \Delta\beta f(\theta_s^n) \max\{S_s^n, 0\} + (1 - \Delta)\beta f(\theta_s^r) \max\{S_s^r, 0\}, \\ w_u &= rU_u - \dot{U}_u = b + \frac{\Delta}{1 + I}\beta[\pi f(\theta_s^n) \max\{S_u^n, 0\} + (1 - \pi)f(\theta_u^l) \max\{S_u^l, 0\}] \\ &\quad + \frac{1 - \Delta}{1 + I}\beta f(\theta_u^r) \max\{S_u^r, 0\} + \frac{I}{1 + I}\beta f(\theta_u^o) \max\{S_u^o, 0\} + \delta(U_s - U_u). \end{aligned} \quad (6)$$

The reservation wage equals the value of leisure, b , plus a share β of the expected surplus at different jobs multiplied by the rate at which the worker obtains these jobs.

The equilibrium tightness for each type of job is determined by free entry:

$$\kappa \geq q(\theta_j^k)(1 - \beta)\mathbb{E}_S[\max\{S, 0\}|k, j] \quad (7)$$

with equality when $\theta_j^k > 0$. Here, $\mathbb{E}_S[\max\{S, 0\}|k, j]$ denotes the expected surplus of a match that is obtained by posting a vacancy v_j^k . The max operator indicates that a firm rejects matches that yield a negative surplus. Given that vacancies in old, regular and stepping-stone jobs are matched to a single type of worker, the expected surplus is $\max\{S_j^k, 0\}$.

Because firms that post novel jobs, $v_s^n(i)$, are matched to both skilled and unskilled workers, their free entry condition becomes:

$$\mathbb{E}_S[\max\{S, 0\}|n, j] = \gamma^n \max\{S_s^n, 0\} + (1 - \gamma^n) \max\{S_u^n, 0\}, \quad (8)$$

with γ^n the probability that vacancies for novel jobs yield a match with a skilled worker (see equation 1). Because $S_s^n > S_u^n$ (see lemma A1 in the Theory Appendix), when the mismatch is severe and firms are pessimistic about finding skilled workers (γ^n is low) they create less novel jobs and reduce tightness. This response constitutes the job creation externality. The externality arises because firms earn quasi-rents in the form of a share of the surplus of the match, and so they care about obtaining workers who yield the largest surplus. If firms paid workers their full marginal product, wages would adjust to reflect the differences in productivity and this mechanism would not operate.

Given a starting value for the state variables $\{x(0), u(0), s(0), \gamma(0), I(0)\}$, an equilibrium consists of an allocation in which the value functions $\{U_s(t), U_n(t), S_s^n(t), S_u^n(t), S_s^r(t), S_u^r(t), S_u^l(t), S_u^o(t)\}$ satisfy the Bellman equations (4), (5) and (6); and the equilibrium tightnesses $\{\theta_s^n(t), \theta_u^l(t), \theta_s^r(t), \theta_u^r(t), \theta_u^o(t)\}$ are determined implicitly by equation (7).

1.2 Analysis of the model

Throughout I assume that $x(0) < 1$ so that not all workers are skilled and the structural change induces a skill mismatch. The Theory Appendix contains the proofs of all the propositions.

I start by analyzing the long-run behavior of the equilibrium. Before the structural change, I assume that $z(0) = 1$ and that the economy is in steady state. By u^*, θ^*, f^* and v^* I

denote the equilibrium unemployment rate, tightness, finding rate and number of vacancies, respectively, in the status quo of this economy. These correspond to the equilibrium objects in the usual search and matching model with no heterogeneity.¹³ Proposition 1 shows that the effects of structural change are only temporary and the economy reverts to its status quo.

PROPOSITION 1 (STEADY-STATE BEHAVIOR) *The economy converges to a unique steady state with $x(t), \gamma(t) \rightarrow 1$, $u(t) \rightarrow u^*$ and $\theta(t) \rightarrow \theta^*$. In this steady $f_s(t) \rightarrow f^*$ and $f_u(t) \rightarrow f_u^*$.*

The economy adjusts as $x(t) \rightarrow 1$ and $I(t) \rightarrow 0$, both because the economy creates stepping-stone jobs and because unskilled workers eventually become skilled at the rate $\delta > 0$. Because structural change does not affect the measure and productivity of jobs available to skilled workers, the economy reverts to its initial status quo over the long run.

To characterize the transitional dynamics, I focus in the case in which gross flows are large. In this case, all state variables but $x(t)$ —the share of skilled workers—and $I(t)$ —the number of available old jobs—adjust immediately and exhibit no propagation on their own, which simplifies the analysis and allows me to derive analytically a clean characterization of the adjustment. Because gross flows are so large in U.S. markets (see Davis and Haltinwanger, 1990) this case is also empirically relevant.

Let $a = \tilde{a}\xi$ and $\lambda = \tilde{\lambda}\xi$, and suppose $\xi \rightarrow \infty$, so that the gross flows between employment and unemployment are large. Because separation rates are large, future reservation wages or productivities do not affect the current surplus of jobs. The normalized surpluses, ξS_j^k , in each job, the reservation wages, w_s and w_u , the finding rates, f_s and f_u , and the tightness θ_u^l , are well defined in this limit and only depend on the current value of $z(t), x(t), I(t)$ and $\Omega(t) = U_s(t) - U_u(t)$ —the incentives to acquire skills (see the Theory Appendix for details).¹⁴

Moreover, the right hand sides of the equations for $\dot{u}, \dot{\gamma}$ and \dot{s} must converge to zero, so

¹³In particular, tightness and unemployment are implicitly defined by the equations $(1 - \beta)(1 - b) = \frac{r + \lambda + \beta\theta^*q(\theta^*)}{q(\theta^*)}\kappa$ and $u = \frac{\lambda}{\lambda + f(\theta^*)}$.

¹⁴Let $\tilde{f} = \tilde{a}\theta^{1-\eta}$ and $\tilde{q} = \tilde{a}\theta^{-\eta}$. Formally, the normalized surpluses are well defined and given by:

$$\begin{aligned} \xi S_s^n &= \frac{z(t) - w_s}{\tilde{\lambda}}, & \xi S_s^r &= \frac{z(t) - w_s}{\tilde{\lambda}}, & \xi S_u^n &= \frac{z(t) - q^n - w_u}{\tilde{\lambda}}, \\ \xi S_u^r &= \frac{z(t) - w_u}{\tilde{\lambda}}, & \xi S_u^o &= \frac{z(t) - w_u}{\tilde{\lambda}}, & \xi S_u^l &= \frac{z(t) - q^n - w_u + \max\{-q^l + \alpha\Omega\}}{\tilde{\lambda}}, \end{aligned}$$

The reservation wages are well defined in the limit, and are given by

$$\begin{aligned} w_s &= b + \Delta\beta\tilde{f}(\theta_s^n) \max\{\xi S_s^n, 0\} + (1 - \Delta)\beta\tilde{f}(\theta_s^r) \max\{\xi S_s^r, 0\}, \\ w_u &= b + \frac{\Delta}{1 + I}\beta[\pi\tilde{f}(\theta_s^n) \max\{\xi S_u^n, 0\} + (1 - \pi)\tilde{f}(\theta_u^l) \max\{\xi S_u^l, 0\}] + \frac{1 - \Delta}{1 + I}\beta\tilde{f}(\theta_u^r) \max\{\xi S_u^r, 0\} + \frac{I}{1 + I}\beta\tilde{f}(\theta_u^o) \max\{\xi S_u^o, 0\} + \delta\Omega. \end{aligned}$$

Finally, the equilibrium tightnesses are given by $\kappa \geq \tilde{q}(\theta_j^k)(1 - \beta)\mathbb{E}_S[\max\{\xi S, 0\} | k, j]$. These equations coincide with a steady state in which z, x, I, Ω are fixed over time.

that $u(t)$, $\gamma(t)$ and $s(t)$ are determined solely by the current value of $z(t)$, $x(t)$, $I(t)$ and $\Omega(t)$:

$$1 - \gamma = (1 - x) \frac{\lambda + \gamma f_s + (1 - \gamma) f_u}{\lambda + f_u}, \quad u = \frac{\lambda}{\lambda + \gamma f_s + (1 - \gamma) f_u}, \quad s = (1 - x) \frac{\frac{\Delta}{1+I} (1 - \pi) f(\theta_u^l)}{\lambda + f_u}. \quad (9)$$

These equations implicitly define $\gamma(z, x, I, \Omega)$, $u(z, x, I, \Omega)$ and $s(z, x, I, \Omega)$, which are independent of their past values and adjust immediately. Here, $1 - \gamma(t)$ tracks $1 - x(t)$, but takes into account the different finding rates of skilled and unskilled workers. Unemployment depends on the average finding rate $f = \gamma f_s + (1 - \gamma) f_u$. The variable $\Omega(t)$ summarizes the incentives to acquire skills, which determine employment in stepping-stone jobs.

To analyze the model, I maintain three assumptions. First, I assume that $q^l > \bar{q}$, with $\bar{q} = (\alpha + r)\Omega^* - q^n$ (here, Ω^* is the steady-state value for $\Omega(t)$). This restriction guarantees that $S_u^n \leq S_u^l < S_s^n$ in equilibrium, so that unskilled workers produce a lower surplus than skilled workers in novel jobs. Second, I assume that $\beta < \bar{\beta}$. This restriction guarantees that the job-finding rate of unskilled workers decreases when $I(t)$ is low and old jobs close. For large values of β , a decline in $I(t)$ lowers unskilled workers' reservation wages so much that firms could end up creating a large number of regular jobs and increasing unskilled workers' finding rates. Third, I assume that $\pi < \bar{\pi}$. This restriction guarantees that the equilibrium is unique and that the externalities do not introduce instabilities in the adjustment of the economy. The thresholds $\bar{\beta}, \bar{\pi} > 0$ are derived in the Appendix. The conditions $\beta < \bar{\beta}$ and $\pi < \bar{\pi}$ are not demanding. For the parametrization of my model introduced in Section 2, any value of $\pi \in [0, 1)$ and values of β as large as 0.9 satisfy these conditions.

The following proposition summarizes the properties of the transitional dynamics.

PROPOSITION 2 (TRANSITIONAL DYNAMICS) *Let $a = \tilde{a}\xi$ and $\lambda = \tilde{\lambda}\xi$, and suppose $\xi \rightarrow \infty$.*

1. *The current values of x, I , and Ω are a sufficient statistic for the equilibrium objects. The behavior of x, I and Ω boils down to the system of differential equations:*

$$\dot{x} = (1 - x) \left[\alpha \frac{\frac{\Delta}{1+I} (1 - \pi) f(\theta_u^l)}{\lambda + f_u} + \delta \right], \quad \dot{\Omega} = r\Omega + w_u - w_s \quad \dot{I} = -v(t)I,$$

coupled with an initial condition for $x(0)$ and $I(0)$, and paths for $z(t)$ and $v(t)$.

2. *The system is globally saddle-path stable and converges to $x(t) = 1, I(t) = 0, \Omega(t) = \Omega^*$. If $z(t) = 1 \forall t$, the stable arm is described by a curve in which $x(t)$ and $\Omega(t)$ increase monotonically to their steady-state values and $I(t)$ declines at the exogenous rate $v(t)$.*

Figure 5 shows the phase diagram for the equilibrium (holding $I(t)$ and $z(t)$ constant). The dotted lines are the loci for $\dot{U} = 0$ and $\dot{x} = 0$ (a vertical line through $x = 1$). Starting from any $x(0)$, the incentives to upgrade skills, $\Omega(0)$, jump to the stable arm and both $\Omega(t)$ and $x(t)$ converge monotonically to the steady state.

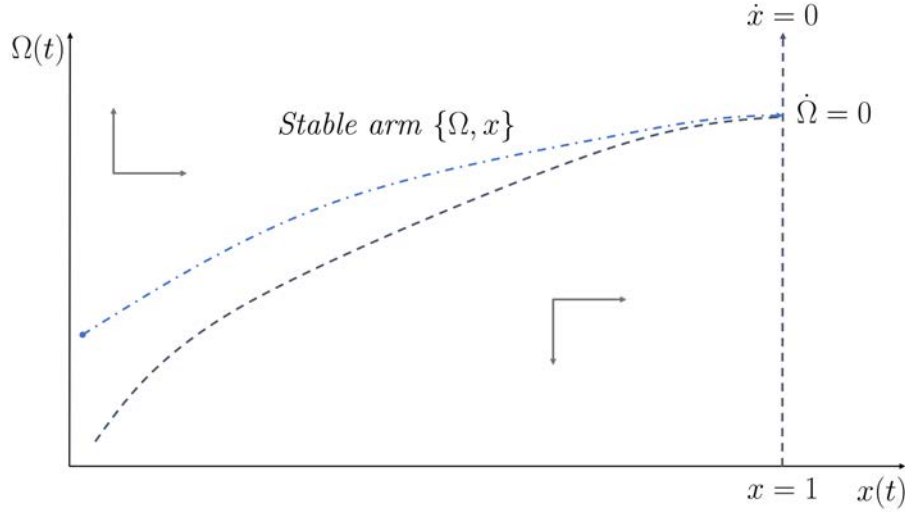


FIGURE 5: Phase diagram for the equilibrium in terms of $x(t)$ and $\Omega(t)$.

When gross flows are large, as they are in the U.S. data, the bulk of the state dependence in my model and the labor market consequences of structural change are driven by the behavior of $x(t)$ and $I(t)$. The dynamics of the remaining state variables introduce minor effects, as is the case in the usual parametrizations of the canonical search model (see Shimer, 2005).

The following proposition characterizes the adjustment when $z(t) = 1, I(t) = 0 \forall t$; there are no aggregate shocks and unemployment is driven by the endogenous behavior of $x(t)$.

PROPOSITION 3 (STRUCTURAL UNEMPLOYMENT) *Suppose $z(t) = 1$ and $I(t) = 0$ for all t . The adjustment to structural change satisfies:*

1. $f_u(t) < f_s(t)$ for all $t \geq 0$.
2. Along the transition, we have that $f_s(t) < f^*$ and $f_u(t) < f_u^*$ for all $t \geq 0$. Moreover, both $f_s(t)$ and $f_u(t)$ increase over time for all $t \geq 0$.
3. A lower $x(0)$ shifts down the entire equilibrium path for $x(t)$, the average finding rate $f(t)$ and the finding rates $f_s(t), f_u(t)$.

The proposition shows that the skill mismatch induced by structural change—captured by the share of unskilled workers, $1 - x(0)$ —causes unemployment along the transition. Unemployment is driven by a decline in the average finding rate $f = \gamma(t)f_s(t) + (1 - \gamma(t))f_u(t)$. The deviation of the average finding rate with respect to its initial level is given by:

$$f - f^* = (1 - \gamma(t))[f_u(t) - f_s(t)] + [f_s(t) - f^*].$$

The first term, $(1 - \gamma(t))[f_u(t) - f_s(t)] < 0$, captures the direct effect of structural change—as I labeled it in the Introduction. A lower $x(0)$ increases the share of unskilled workers

among the unemployed at all points in time, $1 - \gamma(t)$, and these workers have a lower finding rate.¹⁵ The second term, $f_s(t) - f^* \leq 0$ captures the effect of the job creation externality. When $x(0)$ is small, firms anticipate that more unskilled workers will be searching for novel jobs. Firms respond to the skill mismatch by creating less novel jobs (per searcher), which reduces the finding rate of both skilled and unskilled workers below their steady-state levels: $f_s(t) < f_s^*, f_u(t) < f_u^*$. In contrast, in the limit when $\pi = 0$ and there is no random matching we have $f_s(t) = f^*$, and the skill mismatch only increases unemployment via the direct effect.

The proposition also clarifies the nature of unemployment in my model. Despite the fast flows, the skill mismatch—the interplay between a low $x(t)$ and the lack of old jobs—creates unemployment by reducing the average finding rate. Contrary to models of reallocation that build on Lucas and Prescott (1976), the time it takes workers to move from searching for old to new jobs—search unemployment—plays no role in my framework (or at most a minor role if my model is parametrized to match the large gross flows in the data).¹⁶ Matching frictions are important not because of the search unemployment they create but because of the way in which they affect job creation by firms.

Given the large inflow of unskilled workers searching for novel jobs, one would be tempted to conclude that firms could profit from creating a large number of stepping-stone jobs and that the skill mismatch would not last for long. However, through the complementarity effect outlined in the Introduction, the skill mismatch dampens the creation of stepping-stone jobs.

PROPOSITION 4 (COMPLEMENTARITIES IN SKILL UPGRADING) *Suppose $z(t) = 1$. Along the adjustment, we have that $\Omega(t)$ —the incentive of unskilled workers to become skilled—increases over time. Moreover, a lower $x(0)$ shifts the entire equilibrium path for $\Omega(t)$ down.*

The upward-sloping locus for the stable arm in Figure 5 depicts the complementarity effect: for small $x(t)$, the incentives to acquire skills, $\Omega(t)$ are lower, and these increase over time as more workers become skilled.

The complementarity effect results from the fact that skilled workers derive a larger increase in their utility from the novel jobs that are affected by the job creation externality than unskilled workers do. Thus, a worst skill mismatch hurts skilled workers more than it hurts the unskilled and reduces the value of becoming skilled, $\Omega(t)$. In my model, this feature follows from the fact that $S_s^n < S_u^n$ —which reflects unskilled workers lower productivity in

¹⁵Intuitively, this is the case because all workers are matched to novel and regular jobs at some rates, but because of training costs ($q^l > \bar{q}$) and their lower productivity, they face lower finding rates for novel jobs than skilled workers.

¹⁶Pilossoph (2014), too, argues that sectoral reallocation can create little unemployment when gross flows are large. The result for my limit case echoes her findings, and it shows that my theory of unemployment, which is based on the mismatch of skills, is not affected by this criticism.

novel jobs—and the assumption that skilled workers exogenously search more often for these jobs than do unskilled workers. I find this assumption plausible and intuitive. If workers were able to direct their search efforts, and given that $S_s^n < S_u^n$, skilled workers would still search for novel jobs more often than will unskilled workers.¹⁷

The main implication of 4 is that the complementarity effect reduces the creation of stepping-stone jobs, which further amplify unemployment and prolongs the skill mismatch. This occurs because stepping-stone jobs are profitable to the extent that workers are willing to take wage cuts to retrain (see equation 5). During periods of severe mismatch, workers perceive a lower value of acquiring skills. Thus, firms not only post few novel jobs; they do not take full advantage of the large inflow of unskilled workers whom they could retrain.

Propositions 3 and 4 combined imply that, due to the job creation externality and the complementarity effect, unemployment will be accompanied by a drop in tightness and vacancy creation. My model overcomes the critiques of Abraham and Katz (1986) and Blanchard and Diamond (1989) to theories of structural unemployment.

As mentioned in the Introduction, the negative effects of structural change may concentrate in recessions. To explore the interaction between structural change and the business cycle, I characterize the equilibrium of an economy adjusting to structural change which is hit by an unanticipated recession that lasts from time T_i to T_f . I model recessions as bringing two aggregate shocks. First, the recession causes a temporary decline in productivity from $z(t) = 1$ to $z_L < 1$ for $t \in [T_i, T_f]$. In addition, the recession increases the rate at which firms close old jobs to \bar{v} for $t \in [T_i, T_f]$, while $v(t) = v < \bar{v}$ otherwise. The high rate \bar{v} reflects the possibility that firms use recessions to replace old jobs with new technologies, and restructure or close job positions that are at risk of becoming obsolete due to advances in technology, as discussed in the Introduction. The following proposition characterizes the effects of both shocks. To emphasize the business-cycle effects of the recession, I describe my results in terms of the deviations from the trend that would result if there were no recession.

PROPOSITION 5 (INTERACTION WITH A RECESSION) *Consider an economy that is adjusting to structural change in which $x(T_i) < 1$ and $I(T_i) > 0$. Then:*

1. *The decline in productivity reduces both $f_s(t)$ and $f_u(t)$ below their trend for $t \in [T_i, T_f]$. When $x(T_i)$ is small, the average finding rate, $f(t)$, and both finding rates $f_s(t)$ and $f_u(t)$ are more cyclical.*

¹⁷ To substantiate this point, in the Theory Appendix I present an extension of my model in which workers are able to partially direct their search efforts. In this extension, workers allocate their search efforts based on idiosyncratic shocks that garble their expected utility of searching for jobs in each particular task. I derive the equilibrium distribution of workers searching for each job and show that skilled workers allocate a greater share of their time to searching for novel jobs than unskilled workers. I also show that, even if allowed, skilled workers would search for old jobs less than unskilled workers do.

2. For any $T_p > T_f$, there exists a training cost $q(T_p) \in [\bar{q}, \infty)$, such that for $q^l = q(T_p)$ we have that the increase in $v(t)$ reduces both $f_s(t)$ and $f_u(t)$ below their trend for $t \in [T_i, T_p]$. For a given q^l , the reduction in the average finding rate, $f(t)$, and in both $f_s(t)$ and $f_u(t)$ is larger and more long-lasting when $x(T_i)$ is small.

Numeral 1 shows that the temporary fall in productivity reduces both workers' finding rates. The interaction with a small $x(T_i)$ follows by noting that unskilled worker's finding rate is more responsive to changes in productivity. Because the reservation wage of unskilled workers is close to their value of leisure, their wage does not adjust much in response to productivity shocks, but their finding rate does. Due to their low finding rate during recessions, unskilled workers become numerous among the unemployed, which reduces $\gamma(t)$ and exacerbates the job creation externality (see also Pries, 2008). This effect increases the cyclicity of both finding rates f_s and f_u , as well as the average finding rate.

Although this mechanism explains why the finding rate of *both* workers is more cyclical, it does not create any significant source of propagation. When productivity recovers so does the finding rate of both workers. By itself, a temporary productivity shock causes no propagation because it does not affect the behavior of $x(t)$ nor $I(t)$. This observation extends to an environment with heterogeneous agents, the result that productivity shocks create no internal propagation in the canonical search model (Shimer, 2005).

In contrast, Numeral 2 shows that the temporary increase in the rate at which firms close old jobs causes a long-lasting decline in both f_s and f_u . This is so because a low $I(t)$ reduces employment opportunities for unskilled workers and pushes them to redeploy to novel jobs. Following a recession, the inflow of unskilled workers that are searching for novel jobs exacerbates the skill mismatch—an effect that becomes more severe when $x(T_i)$ is small. Firms respond by creating less novel jobs, which reduces both workers finding rates in a persistent manner.

The finding rates f_s and f_u will be depressed until workers retrain and the skill mismatch abates. Numeral 2 of the proposition emphasizes this point and shows that when training costs are high, the effect of the decline in $I(t)$ on job-finding rates outlasts the recession. This mechanism creates a jobless recovery in which the finding rate of both workers remains depressed, relative to their trends, even though productivity has already recovered.¹⁸

The effect of a decline in $I(t)$ on labor markets hinges on the assumption that it affects the finding rate of unskilled workers more than it affects the finding rate of skilled workers. In my model this feature follows from the fact that only unskilled workers search for old jobs.

¹⁸As this discussion clarifies, the closure of old job positions during recessions is different from an increase in the separation rate (as emphasized by Jaimovic and Siu, 2014). An increase in separations contributes to unemployment but it does not affect the state variables $x(t)$ and $I(t)$, and therefore cannot generate propagation.

Though clearly a stark simplification, the general idea that unskilled workers will search more frequently for old jobs than skilled ones seems plausible. After all, unskilled workers lack the skills that are required in other jobs (see also footnote 17). Moreover, both results in Proposition 5 hinge on the assumption that the worsening composition of the unemployment pool affects firm hiring efforts. This would still apply if, while on the job, workers also searched for jobs so long as they do so less frequently than unemployed workers. All the same, my analysis applies to firm’s hiring efforts directed at workers who are currently unemployed, and implies a reduction in the rate at which unemployed workers find jobs.

I derived the results in Proposition 5 under the assumption that $I(t)$ declines exogenously during recessions. Although my empirical findings and Figure 2 support this assumption, it is worth discussing it more thoroughly. The purpose of the assumption is to show what could happen if firms restructured their demand for different types of labor during a recession, without explaining why that could be the case. My results here indicate that it is important to understand when and why do recessions prompt such behavior by firms. In the Theory Appendix I show that one possibility is that, due to the competition from technology, the production of old tasks using labor becomes unprofitable. Firms do not close this vacancies because they made irreversible investments which they have to liquidate or redeploy to the production of other tasks. But liquidating or redeploying investments disrupts current production (see Aghion and Saint Paul, 1998). Thus, firms would endogenously concentrate their liquidation and restructuring efforts during recessions, when the opportunity cost of the foregone production is small. When the recession is over, firms do not create new openings for old jobs because these are unprofitable. This extension generates the same pattern as an exogenous increase in $v(t)$ during recessions.

Proposition 5 has two key implications. First, it suggests that the incidence of skill mismatch rises persistently and lowers job creation both during the recession and the recovery. This feature is consistent with the evidence by Sahin et al. (2014), who show that the incidence of occupational mismatch rose at the onset of the Great Recession. However, using indices of occupational mismatch that are based on the Jackman-Roper condition (see Jackman and Roper, 1987), the literature finds a fast recovery of occupational mismatch after the recession. My model suggests that these indices decline faster than the underlying skill mismatch because, although unskilled workers redeploy to novel jobs—as required by the Jackman-Roper condition—, they lack the requisite skills in these jobs. In my model, the required redeployment of unskilled workers exacerbates the skill mismatch and continues to dampen job creation during the recovery.

Second, the proposition shows that a recession that takes place during periods of structural change produces a different business cycle, which exhibits a larger and more long-lasting increase in unemployment.

I complete my theoretical exploration of the model by characterizing the inefficiencies in the decentralized allocation. This characterization holds for the general case in which a and λ take any positive values.

PROPOSITION 6 (WELFARE ANALYSIS) *Suppose that $\beta = \eta$ and the Hosios condition holds. The constrained efficient allocation has the same structure as the decentralized equilibrium, but the planner values the opportunity cost of workers at μ_s and μ_u given by*

$$\begin{aligned}\mu_s - w_s &= (1 - \eta)\Delta(1 - \gamma^n)f(\theta_s^n)(\max\{S_s^n, 0\} - \max\{S_u^n, 0\}) > 0, \\ \mu_u - w_u &= - (1 - \eta)\frac{\Delta}{1 + I}\pi\gamma^n f(\theta_s^n)(\max\{S_s^n, 0\} - \max\{S_u^n, 0\}) < 0.\end{aligned}\quad (10)$$

Thus, the adjustment of the economy is inefficient. However, the decentralized allocation is constrained efficient in steady state or in the limit case in which $\pi = 0$.

The intuition behind the inefficiency is that, because workers earn quasi rents when they are employed, a reduction in their finding rate has a first-order effect on their utility. Thus, the job creation externality renders the adjustment inefficient.

The Hosios condition internalizes some but not all of the failures in the market. For instance, when $\pi = 0$ and there is no job creation externality, the economy is constrained efficient. In this case, workers who acquire skills are held up by future employers, but this is offset by the congestion these workers create on other skilled workers, as shown by Acemoglu and Shimer (1999). When workers are heterogeneous this reasoning breaks down. Under these circumstances, when a worker becomes skilled he improves matching opportunities for firms that post novel jobs. These firms will be able to extract part of the higher surplus and avoid unskilled matches. This additional external benefit, which translates into more job creation, is not internalized by the Hosios condition (see Shimer and Smith, 2001).

The proposition shows that the planner allocation could be decentralized by taxing search efforts by unskilled workers and subsidizing search efforts by skilled ones (see Shimer and Smith, 2001).¹⁹ The proposition also implies that the returns to training are compressed relative to their social value. When workers become skilled, they reduce the incidence of the job creation externality, but firms and workers do not internalize this social benefit. Subsidizing training increases welfare, as the following corollary shows.

COROLLARY 1 *The social value of skill upgrading exceeds its private counterpart:*

$$\int_t^\infty e^{-r(\tau-t)}(\mu_s - \mu_u)d\tau > \Omega(t).$$

A temporary subsidy to stepping-stone jobs reduces unemployment and increases welfare.

¹⁹This resembles what unemployment insurance and other welfare programs achieve when, as critics argue, they reduce search effort by unskilled takers. Subsidizing old jobs affected by structural change to keep them from becoming obsolete would produce a similar result because it would keep unskilled workers from searching for novel jobs.

2 QUANTITATIVE EXPLORATION

This section explores quantitatively the mechanisms in my model. My numerical exercises also show that the insights derived analytically in the previous section continue to apply when I calibrate gross flows (a and λ) to match the U.S. data.

Table 1 describes a quarterly parametrization of my model. The top panel summarizes standard parameters from the matching literature.²⁰ The bottom panel presents the parameters that quantify the structural change. For these parameters, I define two scenarios: one calibration with $\Delta = 1$ and another with $\Delta = 0.8$. The small values for $1 - \Delta$ reflect the fact that workers displaced by structural change may have few employment alternatives that do not require retraining. In the case of workers displaced from routine-cognitive jobs, service jobs correspond to the main alternative that does not require retraining (see Autor and Dorn, 2013). Despite their growth since 1980, service jobs only employ 13% of workers, and these jobs involve lower wages, which makes them an inviable alternative for many displaced workers. Moreover, during the last 30 years, new professional jobs that are intensive in analytical tasks—reminiscent of novel jobs in my model—account for the bulk of employment growth (see Acemoglu and Restrepo, 2015), which supports my choice of a large value for Δ .

My choice for α is supported by data from *O*NET*, which shows that it takes on average 3 years of training and experience to master a particular occupation or job.²¹ I set $1/\alpha$ to half this value (6 quarters) to account for the possibility that workers productivity increases throughout this period. I also set $1/\delta = 16$, so that on average workers exogenously upgrade their skills every four years. This choice is motivated by the entry of new college cohorts, and the specific value I use plays no role in my results so long as it is small and positive.

In the first scenario, I calibrate values of $q^n = 0.05$ and $q^l = 0.45$ to match estimates for the wage and earning losses for an unskilled worker displaced from an old job (see Davis and von Wachten, 2011). The literature estimates that 15 years after losing a job, a displaced worker’s earnings are 10% lower than his previous income—which informs my choice for q^n —, and the present discounted value of the losses amounts to a full year of his income—which informs my choice of q^l . For these parameters, Figure 6 presents the paths for earnings, wages

²⁰I set the elasticity of the matching function, η , to 0.5 following Pissarides (2009) and the evidence in Mortensen and Petrongolo (2000). I also impose the Hosios condition $\beta = \eta$. In this case with random matching the usual argument that justifies this assumption does not apply (see Shimer 2005). Instead, I assume the Hosios condition to isolate the role of the job creation externality from the other well-known inefficiencies present in matching models. I target quarterly data and set $z = 1$, $a = 1.3$, $\kappa = 0.235$, $b = 0.7$, $\lambda = 0.1$; which guarantee in steady state $\theta^* = 1$ —a normalization—, a quarterly finding rate of 1.3 and a unemployment rate of 7% in steady state. Finally, I set the quarterly discount rate to $r = 0.012$.

²¹Among 729 occupational groups in the *O*NET* data, workers require on average 2.98 years (standard deviation=2.45) of vocational training, plant training or job-related experience to master each occupation.

TABLE 1: Quarterly parametrization of the model.

	Scenario 1	Scenario 2	Source or target
<i>Search model Parameters:</i>			
Steady-state productivity, $z(t)$	1	1	Normalization.
Discount rate, r	0.012	0.012	From Pissarides (2009).
Matching function elasticity, η	0.5	0.5	Mortensen and Petrongolo (2000).
Workers' value of unemployment, b	0.7	0.7	From Pissarides (2009).
Matching function scale, a	1.3	1.3	Quarterly rate from Shimer (2005).
Flow cost of vacancies, κ	0.235	0.235	Normalization $\theta^* = 1$.
Separation rate λ	0.1	0.1	Quarterly rate from Shimer (2005).
Workers' bargaining power, β	0.5	0.5	Hosios condition.
<i>Structural change parameters:</i>			
Share of novel jobs, Δ	1	0.8	Employment growth in high-skill jobs.
Learning rate in stepping-stone jobs, α	1/6	1/6	Half the average time required to master occupations in $O * NET$.
Exogenous replacement rate δ	1/16	1/16	Replacement by new college cohort.
Lower productivity in novel jobs, q^n	0.05	0.05	Wage losses for displaced workers (Davis and von Wachten, 2011)
Training costs, q^l	0.45	0.3	Earning losses for displaced workers (Davis and von Wachten, 2011)
Random matching, π	0.5	0.5	Assumed.

Notes: The table presents the value of the parameters used in my numerical exercises. The columns labeled Scenario 1 and Scenario 2 present the two alternative scenarios I explore.

and employment (relative to their pre-displacement level) for an unskilled worker displaced from an old job at time 0. Expected earnings are 10% lower 15 years after and the present discounted value of the earning losses amounts to 1.06 times the worker's yearly earnings.²²

For the second scenario with $\Delta = 0.8$, I scale q^l down to 0.3, which keeps the surplus of stepping-stone jobs at a level that is similar to that of the first scenario—roughly 0.32 in steady state. The purpose of this scenario is to investigate how the availability of regular jobs that require no skills affects the adjustment of the economy.

Finally, I assume $\pi = 0.5$, so that there is an intermediate but large degree of random

²²This coincides with the cost of unemployment spells estimated by Davis and von Wachten (2011) during periods with low aggregate unemployment. To match this setting in my model, I estimate the earning losses for a single unskilled worker assuming that the tightness of all labor markets is at its steady state level. The large value of q^l implies unskilled workers upgrade their skills at a low rate, matching the persistent losses in earnings and wages in the data. The small positive value of q^n implies unskilled workers may be able to obtain novel jobs without upgrading their skills for several years, but at a slightly lower wage than what they earned before. A larger value of q^n implies a counterfactual sharp drop in earnings followed by a rapid recovery.



FIGURE 6: Earnings, wages and employment for a displaced unskilled worker. Results for Scenario 1.

matching. Larger values of π exacerbate the externalities in my model.²³

2.1 Numerical results

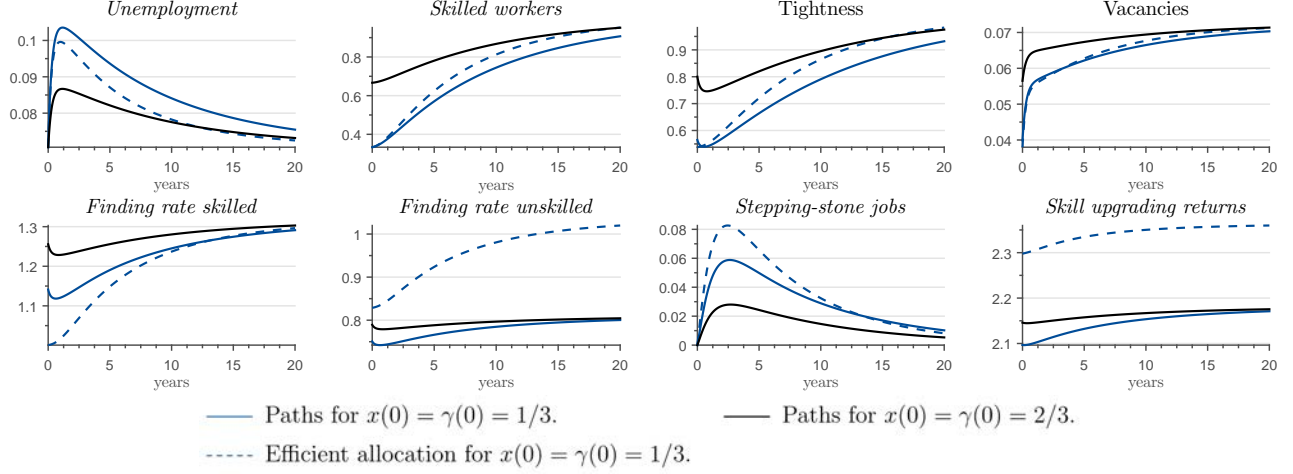
I start by computing the equilibrium adjustment to structural change when $I(0) = 0$, so that no old jobs are available. Figure 7 depicts the equilibrium, which presents the results for the first scenario with $\Delta = 1$ in the top panel and for the scenario with $\Delta = 0.8$ in the bottom panel. In both cases, the blue lines present the equilibrium paths for an economy with $x(0) = \gamma(0) = 1/3$, so that a third of the workers are skilled, and the black lines present the paths for an economy with $x(0) = \gamma(0) = 2/3$. In addition, I set $u(0) = u^*$ and $s(0) = 0$.

When old jobs close immediately, structural change creates a large and long-lasting increase in unemployment. In the first scenario with $\gamma(0) = 1/3$, structural change raises the unemployment rate by 3.2 percentage points in the short run and 1.5 percentage points 10 years thereafter. Unemployment is accompanied by low tightness and few vacancies, which shows that my model overcomes the Abraham and Katz' (1986) critique; during periods of structural change, vacancies and unemployment trace a downward sloping Beveridge curve.

The increase in unemployment is driven by the 30% lower finding rate among unskilled workers—the direct effect—and by the 17% decline in skilled workers' finding rate (relative to its steady state level $f^* = 1.3$) that is caused by the job creation externality. Figure 8 decomposes the unemployment rate for the first scenario and for $x(0) = \gamma(0) = 1/3$. The solid line depicts the unemployment rate. The dashed line shows the unemployment rate that would prevail if both workers' finding rates were set at their steady-state levels, thus removing the job creation externality. The dotted line shows the additional reduction in unemployment that would result if the incentives to acquire skills along the transition were given by Ω^* instead of $\Omega(t)$, which removes the complementarity effect. Although this is one of several

²³The values of π and β used satisfy the restrictions derived for the particular case in which $a, \lambda \rightarrow \infty$. Moreover, the condition $q^l > \bar{q}$ is satisfied. In the first numerical scenario, I have $\bar{q} = 0.34$, which is smaller than $q^l = 0.45$. In the second numerical scenario, I have $\bar{q} = 0.19$, which is smaller than $q^l = 0.3$.

SCENARIO 1, $\Delta = 1$



SCENARIO 2, $\Delta = 0.8$

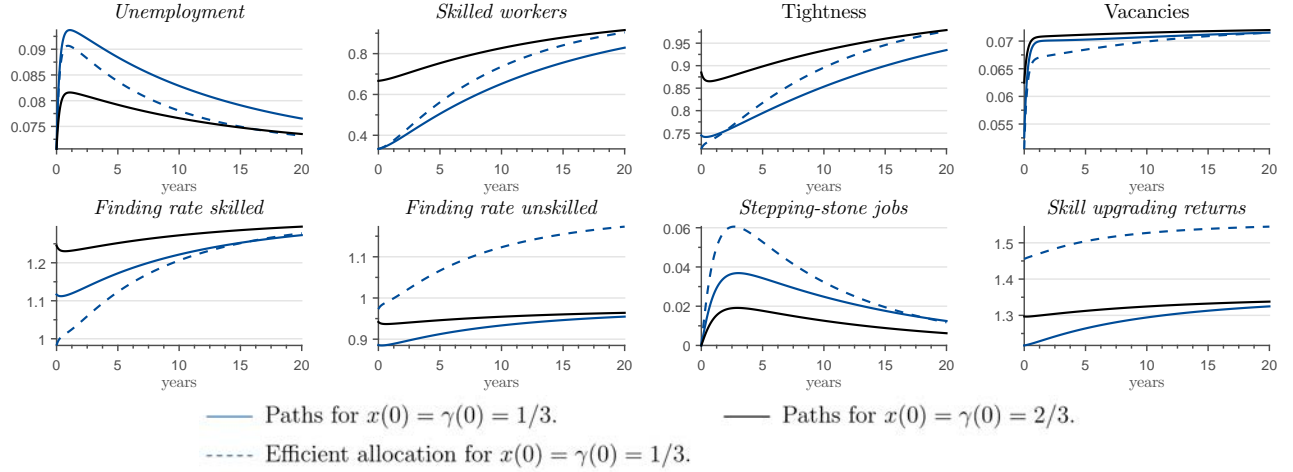


FIGURE 7: Equilibrium adjustment paths for different variables in my model in both scenarios.

possible decompositions, it shows that the job creation externality and the complementarity effect may explain up to 40% of the increase in unemployment.

The market failure is quantitatively relevant. This can be seen from a comparison of the market equilibrium with the paths for the constrained efficient allocation for $x(0) = \gamma(0) = 1/3$ in the dotted blue lines in Figure 7. The constrained efficient allocation involves about 30% less unemployment along the transition and a faster adjustment that is driven by the creation of 50% more stepping-stone jobs in the first years of the adjustment. The figures also show that the private value of acquiring skills is about 10 to 15% smaller than its social value.

This is surprising given that in the calibration used search frictions create only a small wedge between wages and the marginal product of labor. In particular, workers earn a share $\frac{r+\lambda+a\theta^{1-\eta}}{r+\lambda+\eta a\theta^{1-\eta}}\eta \in [90\%, 93\%]$ of the *gross value* of a match, which implies that they are effectively bargaining with firms over small rents. The inefficiencies are large despite the small matching

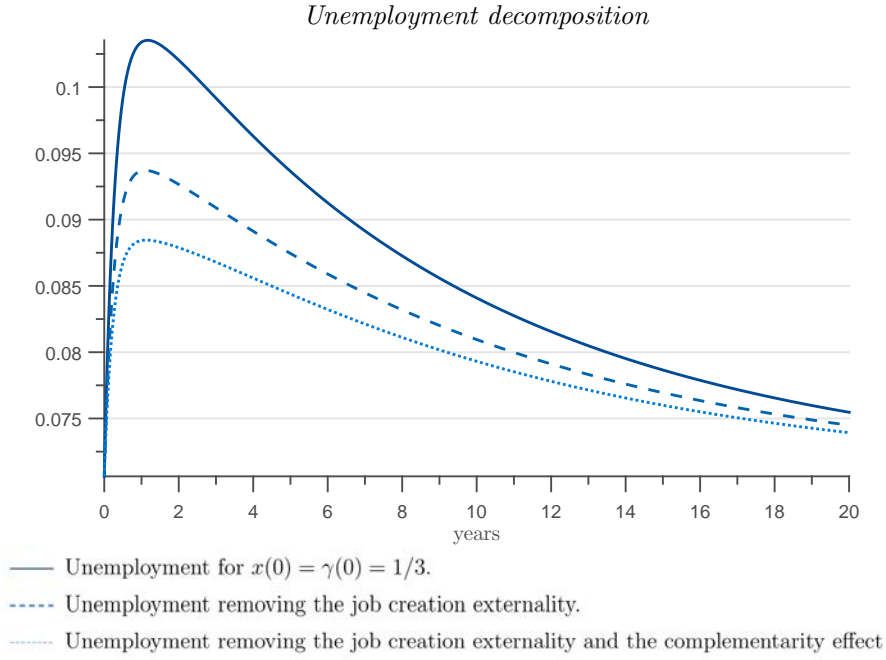


FIGURE 8: Unemployment rate decomposition computed for Scenario 1 in the case with $x(0) = \gamma(0) = 1/3$.

frictions for two reasons. First, rents determine job creation decisions. Even if these rents are small, changes in the frequency at which firms that post novel jobs match with skilled or unskilled workers cause large changes in the creation of novel jobs. Thus, the job creation externality is large (as my decomposition in Figure 8 confirms), and this introduces a wedge between the private and social value of retraining of about 10-15%. Second, when the surplus of stepping-stone jobs is small—as in my calibrations—, a small change in the value of retraining can have a large effect on the number of stepping-stone jobs that are created. In this case, the wage paid to unskilled workers in stepping-stone jobs is close to their outside option and becomes endogenously rigid. A decline in the gross value of stepping-stone jobs—driven by workers’ willingness to acquire skills—results in large changes in quantities instead of wages.

A complementary intuition is that, due to the large training costs, the quasi-supply of unskilled labor in stepping-stone jobs is very elastic, as is shown in Figure 9. In addition, because of the complementarity effect, the demand curve for stepping-stone jobs (i.e., their flow value) is upward sloping in equilibrium. Both forces imply that a small change in the gross value of these jobs creates a large increase in quantities. If the surplus of stepping-stone jobs were large, there would still be an externality. But because the planner would face a much inelastic quasi-supply of unskilled labor, it would not create many additional stepping-stone jobs in response. The inefficiencies would be reflected in prices and not in quantities, and the welfare cost (shaded in gray in the figure) would be smaller than in my calibration.

Finally, I ask whether my model generates a large interaction between an underlying structural change and recessions. For both scenarios in Table 1, I consider an economy that is

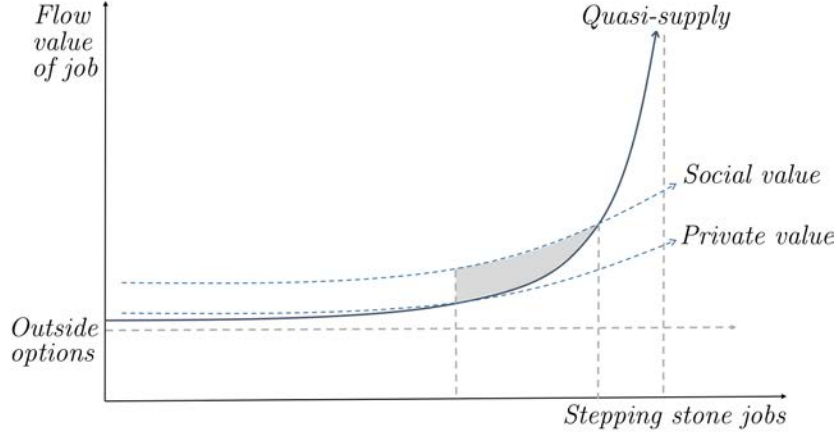


FIGURE 9: Representation of the equilibrium employment in stepping-stone jobs.

adjusting to structural change and I compute its response to an unanticipated recession that takes place 5 years into the adjustment (so that $T_i = 20$). I assume initially $\gamma(0) = 1/5$ and $I(0) = 1$ so that the recession hits the economy when $\gamma(T_i) \approx 1/3$ and $I(T_i)$ is still large. The recession lasts for 10 quarters and reduces labor productivity by 5%, which matches the available estimates for factor productivity during the Great Recession. I set $v = 0.01$ so that old jobs become obsolete at a small secular rate, and I calibrate $\bar{v} = 0.09$ to match the permanent decline in old job openings of roughly 55% depicted in Figure 2.

Figure 10 presents the deviations in unemployment from its level at T_i for both scenarios, as well as the equilibrium path for γ^n . For simplicity, I normalize the starting time of the recession to zero in the figures so that productivity fully recovers by 2.5 years. The gray dotted line presents the (almost negligible) trend in unemployment in the absence of a recession. The slow decline in $I(t)$ guarantees the absence of a trend. The red line presents the behavior of unemployment in the recession. As stated in Proposition 5, when the economy is adjusting to structural change, the recession creates a large and long-lasting increase in unemployment. In the first scenario, unemployment increases by 2.75 percentage points above its trend during the crisis and it remains 1 percentage point above its initial level (and trend) 5 years after the recession ends. In both scenarios, the share of skilled workers among those who are searching for novel jobs, γ^n , falls in a persistent manner during the recession, which shows how the crisis exacerbates the skill mismatch.

As emphasized in my theoretical analysis, the permanent decline in old jobs has a long-lasting effect because it exacerbates the skill mismatch. The skill mismatch lasts because firms and workers do not take full advantage of the opportunity to retrain workers. For comparison, Figure 10 presents the constrained efficient allocation in green. In this allocation, firms and workers engage in the efficient amount of retraining, the increase in the skill mismatch abates

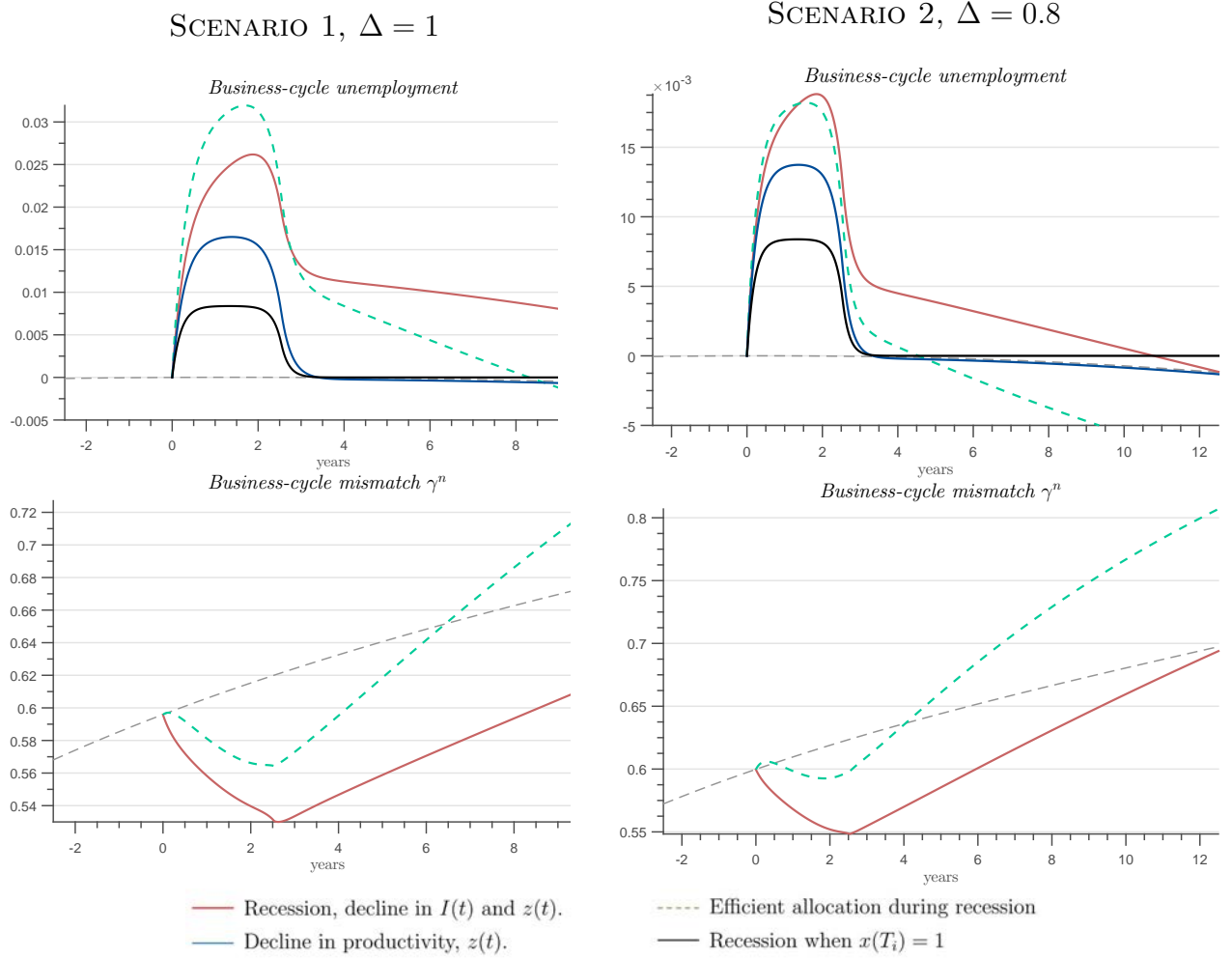


FIGURE 10: Adjustment paths for unemployment relative to its initial value at time 0. The gray line plots the counterfactual trends in an economy adjusting to a structural change that started at $t = -5$. The recession affects the economy from $t = 0$ to $t = 2.5$ (in years)

shortly after the recession ends, and there is little propagation of unemployment.²⁴ However, in the efficient allocation, unemployment may be larger during the onset of the crisis. This occurs because the planner keeps skilled workers searching for jobs to compensate for the more volatile finding rate of unskilled workers. By doing so, the planner maintains a more favorable composition of the unemployment pool, which reduces the job creation externality.

The black line in Figure 10 presents the response of unemployment in an economy that is not experiencing structural change. In line with Shimer's (2005) findings, unemployment only increases slightly (by less than a percentage point) and the finding rate recovers fully by the end of the recession; there is no propagation. The blue line presents the response of unemployment to *only* the decline in productivity in an economy that is adjusting to structural change. In this case, unemployment is amplified during the crisis, and it becomes about two

²⁴The unemployment rate goes below its initial trend because the reduction in old jobs causes workers to upgrade their skills at a faster rate than what they would otherwise do.

times more volatile than in an economy that is not affected by structural change. But as anticipated, in this case, too, there is no significant propagation beyond T_f .

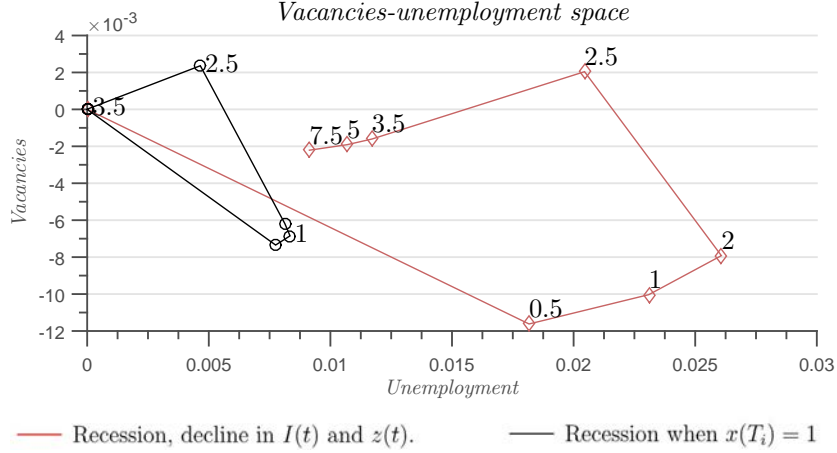


FIGURE 11: Beveridge curves. Both figures center around the initial unemployment and the number of vacancies prior to the recession. Each point corresponds to a different year since the onset of the recession.

Finally, my model matches two salient facts of recessions. First, as Figure 11 shows, when the recession interacts with a structural change the economy recovers through a more pronounced and sluggish counter-clockwise trajectory in the vacancy-unemployment space, as was observed in recent recessions (see Barlevy, 2011 and Veracierto, 2011). The black line shows that the adjustment is less pronounced and faster for an economy not undergoing any structural change. Second, in keeping with the evidence, my model predicts that unemployment spells are more costly during a recession (see Davis and von Wachten, 2011). Due to the lack of old jobs and the skill mismatch, displacement costs unskilled workers an additional 18% loss in earnings when it occurs during a recession.

3 EMPIRICAL IMPLICATIONS

I study the implications of my model in the context of a pervasive structural change affecting U.S. labor markets: the decline in routine-cognitive jobs outside manufacturing. These jobs are the empirical analog of old jobs in my model. I start by documenting the nature of this structural change and the skill mismatch it brought. Then I estimate its effects on employment, unemployment and wages. I finalize by exploring if there was an interaction between this structural change and the Great Recession.

Using data on 330 occupational groups that are consistently defined over time and include all non-military jobs in the U.S. Census, I define routine-cognitive jobs on the basis of indices of task content for each occupation provided by Autor and Acemoglu (2011). I label as routine-cognitive jobs those in the occupational groups among the top tercile that have the

highest routine-cognitive content. These jobs comprise precise and repetitive tasks, which are typical of middle-skill office jobs. The occupations with the highest indices are telephone operators, payroll, postal and time-keeping clerks, and bank tellers.

3.1 The decline in routine-cognitive jobs

In this subsection I document the shift in the occupational structure created by the decline in routine-cognitive jobs. I focus on the time period from 2007 to 2013 because my model suggests the effects of this structural change are stronger during the Great Recession.

I start by estimating the reallocation patterns of workers who specialized in routine-cognitive jobs. To do so, I partition the working-age population in the Census into skill groups that are defined by their demographic characteristics. This yields 200 groups that are defined by sex (male, female), age (16-24, 25-34, 35-44, 45-54 and 55-64 years), education (less than high school, high school, some college, completed college and more than college), and region of residence (Midwest, North, South and West). For each skill group, I use the 2000 Census to compute the share of workers outside manufacturing who were employed in routine-cognitive jobs. This procedure yields a measure of the specialization of each skill group in routine-cognitive jobs, GRC_g . Workers in skill groups that specialized in routine-cognitive jobs are the empirical analog of the unskilled in my model; computers and new technologies depreciated the value of their skills and forced them to reallocate to other jobs as I will show.²⁵

The focus on jobs outside manufacturing separates routine-cognitive jobs from other routine jobs in manufacturing. The latter declined mostly from 1980 to 2000 and brought different patterns of reallocation (see Autor, Dorn and Hanson, 2015). Moreover, during crisis, routine jobs in manufacturing are subject to the large volatility of the manufacturing sector, which confounds sectoral and aggregate shocks (see Foote and Ryan, 2014).

Using data from the American Community Survey, I estimate the equation:

$$\Delta Y_{gt} = \beta GRC_g + \Theta X_g + \varepsilon_{gt} \quad (11)$$

The dependent variable ΔY_{it} is the change between 2007—before the start of the Great Recession—and year t in the share of workers at different occupational categories, Y_{it} . I measure these shares using the American Community Survey; they include both workers who are currently employed in each occupation and unemployed workers who were last employed in each occupation. The categories include service, routine, professional, and managerial jobs. Service, professional and managerial jobs have the least routine content, and they are not

²⁵Underlying my approach are two assumptions: that employment shares in 2000 reflect each skill group's inherent abilities for different jobs; and that these abilities persist over time. I chose the year 2000 as the baseline because it precedes the sharp decline in routine-cognitive jobs experienced in the U.S. in recent years.

directly affected by the computerization of routine tasks. The coefficient β measures the *differential change* in employment for workers who specialized in routine-cognitive jobs.

On the right-hand side of equation (11), X_g are group characteristics measured using the 2000 Census. These include the share of employment in manufacturing, durable, tradable and construction industries, which takes into account differences in industry cyclicity and other sectoral shocks.²⁶ I also include a full set of gender, age, education, and region of residence dummies so that my estimates do not capture differences in formal education, female participation in the labor force or life-cycle dynamics.²⁷

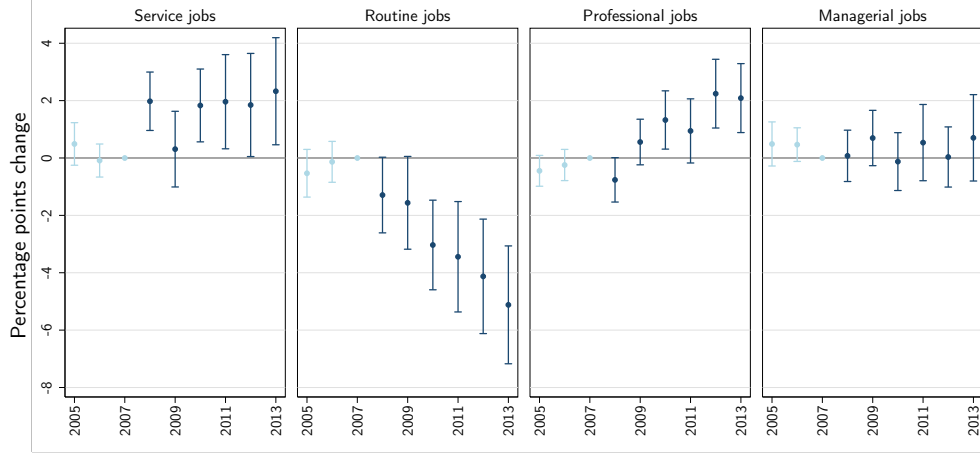
Panel A in Figure 12 shows that, during the Great Recession and its recovery, workers in skill groups that specialized in routine-cognitive jobs redeployed to service and professional occupations. The figure plots year-by-year estimates of equation (11) that are scaled so that they reflect the difference in reallocation between workers at the 90th percentile of specialization in routine-cognitive jobs and workers at the 10th percentile. From 2007 to 2013, among the skill groups that were the most specialized in routine-cognitive jobs, employment in these jobs fell by 5 percentage points relative to the least specialized skill groups. The decline was matched by a redeployment to professional jobs (2 percentage points increase) and service jobs (2 percentage points increase). The reallocation concentrated during the Great Recession and its recovery; it did not occur before, from 2005 to 2007, as is shown by the estimates in light blue for these years.

The same patterns emerge when I move from the national-level data on skill-groups and zoom in at the local labor market. To do so, I use data for 722 commuting zones, which cover the entire continental U.S. territory. For each commuting zone, I compute from the 2000 Census the share of workers outside manufacturing who were employed in routine-cognitive jobs. This procedure yields a measure of specialization in routine-cognitive jobs at the commuting-zone level, RC_i , which I depict in Figure 13. The variation in RC_i stems from historical patterns of specialization as I show in detail below. I think of RC_i as the empirical analog of the share of unskilled workers and the availability of old jobs in each local economy. Commuting zones that have high values of RC_i are the most exposed to the decline in routine-cognitive jobs and they experience the largest change in productive structure. As Autor and Dorn (2013) have shown, in recent decades these commuting zones saw a fast adoption of computers and information technologies, as cheaper computers allowed firms to replace labor in many of the repetitive tasks that comprise routine-cognitive jobs.

²⁶I use data from the County Business Patterns aggregated to the commuting zone level to compute these employment shares. The definition of tradable industries follows Acemoglu et al. (2014), who include agriculture/forestry/fishing, mining, manufacturing and wholesale trade.

²⁷When estimating this equation, I allow the error term ε_{gt} to be correlated within skill groups and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight groups by their size in 2000.

PANEL A: GRC_g MEASURED USING THE 2000 CENSUS.



PANEL B: GRC_g MEASURED USING THE 1980 CENSUS.

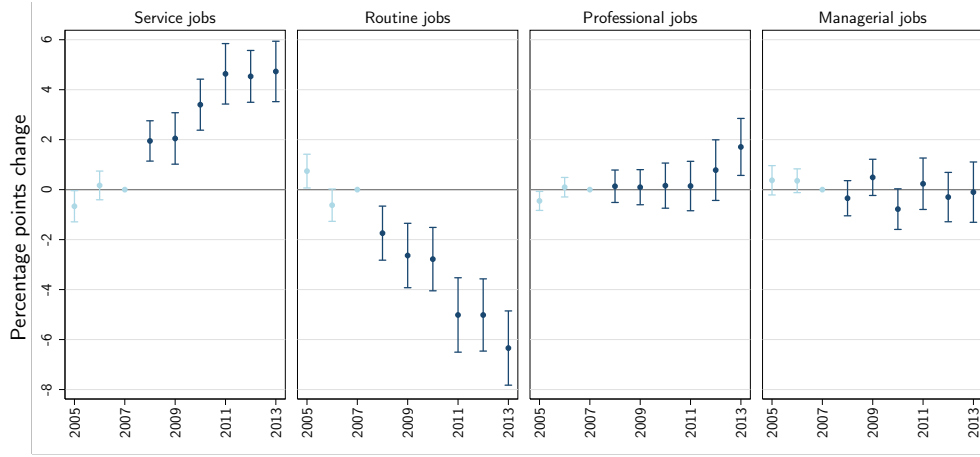


FIGURE 12: Estimated change in the share of workers in service, routine, professional and managerial jobs and their 90% confidence intervals. The estimates compare skill groups at the 90th percentile of specialization in routine-cognitive jobs relative to skill groups at the 10th percentile (2007 is the base year). Each panel indicates the occupational category.

To test if exposed commuting zones saw a larger shift in their productive structure, I estimate the commuting-zone level analog of equation (11):²⁸

$$\Delta Y_{it} = \beta RC_i + \Gamma X_i + \varepsilon_{it} \quad (12)$$

The covariates X_i include a full set of Census Division dummies as well as the year 2000 share of employment in manufacturing, durable, tradable and construction industries, which takes into account differences in industry cyclicity and other sectoral shocks. I also control for

²⁸When estimating this equation, I allow the error term ε_{it} to be correlated within States and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight commuting zones by the size of their workforce in 2000.

Share of employment in routine-cognitive jobs from the 2000 Census

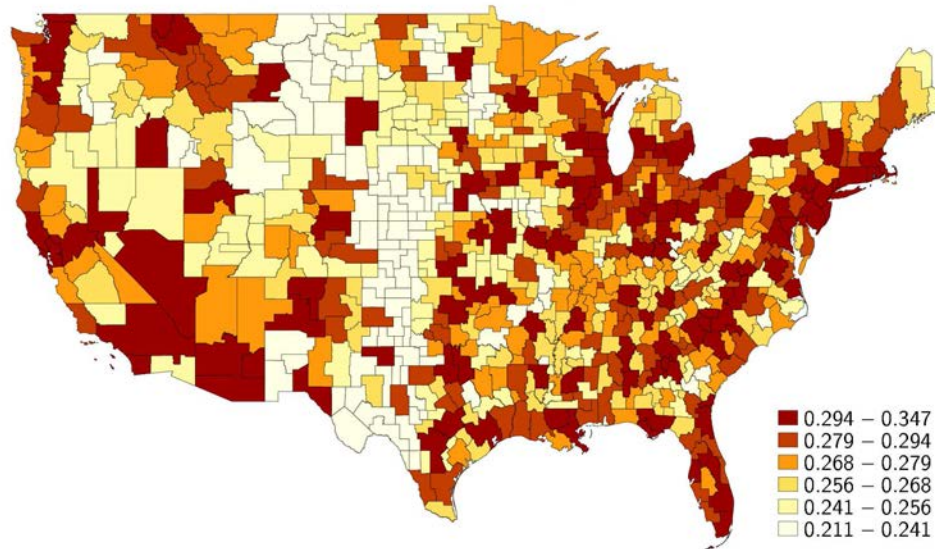


FIGURE 13: Geographical distribution of the exposure of U.S. commuting zones to the decline in routine-cognitive jobs.

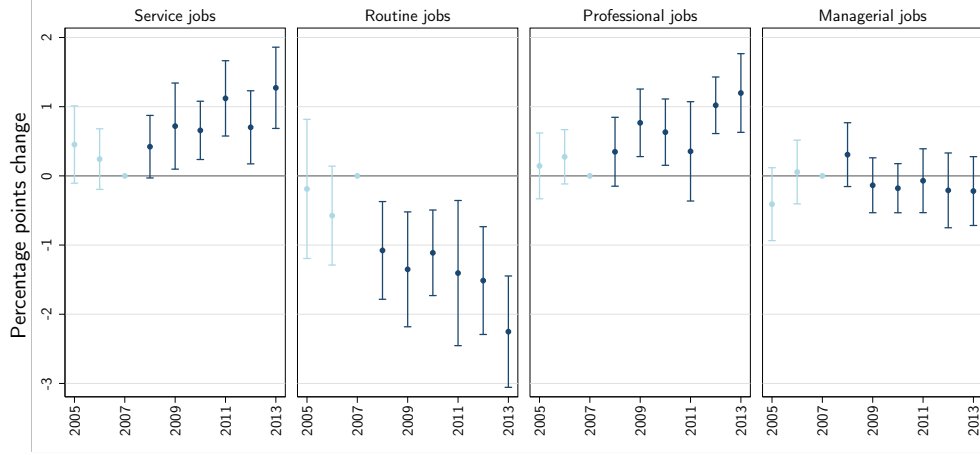
observable characteristics of the workforce, as measured in the 2000 Census, which determine changes in the labor supply. These characteristics include the share of the population of different races, the share of the population that is older than 65 years, the share of foreign workers, the share of workers who have different levels of schooling, the female labor force participation and the share of workers who earn the minimum wage.

Panel A in Figure 14—which is the analog of Figure 12—plots year-by-year estimates of equation (12), which are scaled to reflect the difference between commuting zones at the 90th percentile of exposure to this structural change and zones at the 10th percentile. The figure reveals that, during the great recession, the most exposed commuting zones experienced an increase in job polarization (relative to other zones) that did not take place before.

The findings in Figures 12 and 14 describe the nature of this structural change and underscore the parallels with my model. Due to the sharp decline in routine-cognitive jobs that concentrated in the Great Recession, workers redeployed to both service jobs—the analog of regular jobs in my model—and professional jobs—the analog of novel jobs in my model.

As subsumed in my model, the redeployment of routine-cognitive workers to professional jobs caused a skill mismatch because professional jobs require additional skills and training. Several pieces of evidence support this view. Data from *O*NET* shows that professional jobs are among the most intensive in analytical tasks, while workers in these jobs perform few routine tasks. Workers who specialized in routine-cognitive jobs may lack these analytical skills. Data from *O*NET* also suggests that professional jobs have stringent training requirements: they require on average 2.5 years of training and experience, unlike routine-cognitive jobs, which require only 1.5 years. Figure 15 shows that, among workers who specialized in

PANEL A: RC_i MEASURED USING THE 2000 CENSUS.



PANEL B: RC_i MEASURED USING THE 1980 CENSUS.

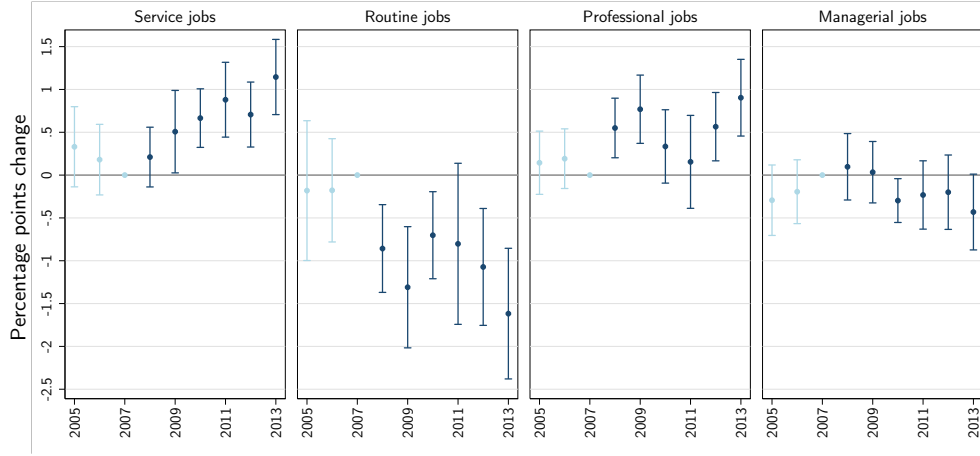


FIGURE 14: Estimated change in the share of workers in service, routine, professional and managerial jobs and their 90% confidence intervals. The estimates compare commuting zones at the 90th percentile of exposure to structural change relative to commuting zones at the 10th percentile (2007 is the base year). Each panel indicates the occupational category.

routine-cognitive jobs, and in exposed commuting zones, the adjustment to this structural change required the reallocation of workers to the top tercile of jobs with the most stringent training requirements. The increase in retraining took place during the Great Recession but not before.

Professional jobs also differ in other dimensions. In 2000, 94% of the workers in professional jobs had some college education, but only 64% of the workers in routine-cognitive jobs did. This, too, points to a mismatch in educational requirements. Finally, data examined by Lin (2013) show that, according to the 2000 Census, 12% of professional jobs corresponded to new job titles. The novelty of these jobs suggests that, particularly among those specialized routine-cognitive job, many of the requisite skills were not commonly held by workers.

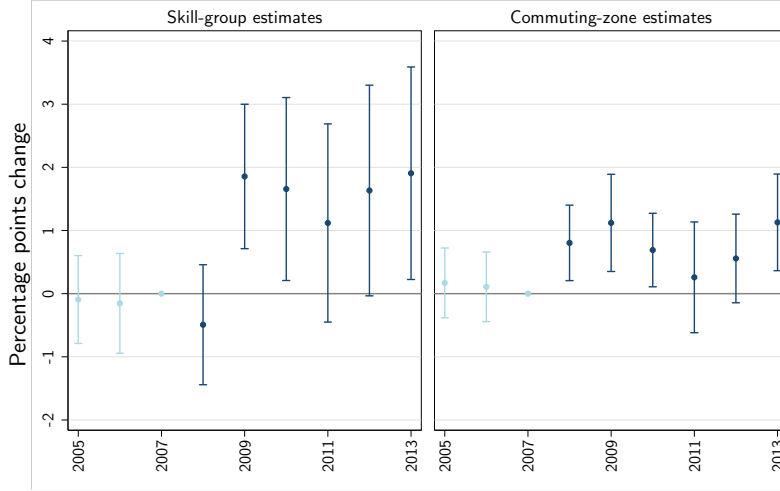


FIGURE 15: Estimated change in the share of workers reporting jobs in the top tercile of occupations with the highest training requirements and their 90% confidence intervals. The left panel presents estimates at the commuting-zone level, and the right panel presents estimates at the skill-group level. Both figures compare the effects at the 90th percentile of exposure to structural change relative to the 10th percentile.

The redeployment of workers deteriorated the expected quality of new hires in professional jobs. In additional results not reported here, I find that the composition of workers employed in professional jobs deteriorated during the Great Recession but not before. The rate at which workers who had no college education were hired in professional jobs increased by 30%. The rate at which professional jobs hired workers from skill groups that in 2000 were not specialized in these jobs also increased. This was driven by a 4% decline in the rate at which professional jobs hired workers from skill groups that in 2000 were specialized in analytic tasks, and a 10% increase in the rate at which professional jobs hired workers from skill groups that in 2000 were specialized in routine-cognitive tasks.

The findings in this section do not result from a mechanical decline in the share of workers *currently employed* in routine jobs or from mean reversion. If workers did not reallocate, they would still report they were last employed in routine-cognitive jobs, and the patterns in the figures would not emerge.²⁹ Contrary to what I find in the data, mean reversion would show up strongly before the Great Recession. Moreover, Panels B in both Figures present my estimates when I measure RC_i and GRC_g using employment shares from 1980. Although mean reversion should not play any significant role in this case, I obtain similar findings.³⁰ Finally,

²⁹The ACS reports the occupation held by non-employed workers in their last job, provided that they had a job in the last 5 years. Thus, it is unlikely that these facts are explained by differences in attrition.

³⁰For the period from 1990 to 2000, I estimate an annual convergence rate of 0.0429 percentage points per year (standard error=0.0275) in the share of workers employed in routine-cognitive jobs among areas with a 10 percentage points additional share in 1980. For the period from 1980 to 1990, I estimate an annual convergence rate of 0.01 percentage points per year (standard error=0.003) in the share of workers employed in routine-

in results presented below, I find that local demand shocks in a commuting zone prompted a faster decline in routine-cognitive employment, which weighs in favor of my interpretation of the findings in this subsection.

3.2 Estimating the effects of the skill mismatch

The key empirical implication of my model is that, due to the job creation externality, the finding rate of a worker not only depends on his skills, but also on the extent of the skill mismatch in his labor market. The mismatch results from the large redeployment of displaced workers to professional jobs during the Great Recession, which causes a reduction in professional-job openings.

Let f_{igt} be the finding rate for workers from skill group g who reside in commuting zone i , and let Δf_{ig} be the change in their finding rate that is caused by structural change during the Great Recession. My theoretical analysis establishes that, for the case of two skill groups $g \in \{u, s\}$ we can approximate the change in their finding rate as:

$$\Delta f_{ig} \approx \frac{\partial f_s}{\partial z} \Delta z + \mathbb{1}\{g = u\} \left(\frac{\partial f_u}{\partial I} \Delta I + \left(\frac{\partial f_u}{\partial z} - \frac{\partial f_s}{\partial z} \right) \Delta z \right) + \frac{\partial f_g}{\partial \gamma} \Delta \gamma_i^n.$$

The first term $\frac{\partial f_s}{\partial z}$ captures the effect of a lower productivity during the Great Recession, $\Delta z < 0$, that is common to all workers. The second term $\frac{\partial f_u}{\partial I} \Delta I + \left(\frac{\partial f_u}{\partial z} - \frac{\partial f_s}{\partial z} \right) \Delta z < 0$ captures the direct effect of the structural change and the recession, which is specific to all unskilled workers. For instance, unskilled workers face lower finding rates because there are less old jobs (recall that $\Delta I < 0$ during recessions) and aggregate productivity is lower (recall that the finding rate of unskilled workers is more cyclical).

My model underscores the role of the additional term $\frac{\partial f_g}{\partial \gamma} \Delta \gamma_i^n$, which captures the role of the job creation externality and the complementarity effect on all workers' finding rates. This term captures the worsening of the skill mismatch during the Great Recession, which is quantified by $\Delta \gamma_i^n < 0$ and which reduces the creation of novel and stepping-stone jobs.

This equation motivates the following regression model:

$$\Delta f_{igt} = \delta_t + \beta^d GRC_g + \beta^e RC_i + \Theta X_g + \Gamma X_i + \varepsilon_{igt}. \quad (13)$$

Here, δ_t parametrizes the common effect of aggregate shocks. The term $\beta^d GRC_g$ parametrizes the direct effect, which depends only on workers' skills. I proxy workers' skills using the measure of specialization in routine-cognitive jobs, GRC_g .

cognitive jobs among areas with a 10 percentage points additional share in 1980. Both estimates imply a level of mean reversion that could explain at most a tenth of the documented decline in routine-cognitive jobs from 2007 to 2013 in highly exposed commuting zones. Similar findings apply for skill groups.

The term $\beta^e RC_i$ parametrizes the effect of the worsening skill mismatch in a local labor market. As shown in Figure 14, during the Great Recession, commuting zones with high routine-cognitive employment experienced a large redeployment of displaced workers to professional jobs. Thus, in commuting zones with a high RC_i , the adjustment of the economy during the recession created a severe skill mismatch—a sharp decline in $\Delta\gamma_i^n$. In commuting zones with low routine-cognitive employment, the number of routine-cognitive workers that redeploy to professional jobs is small in comparison to the number of workers who already possess the skills required in these jobs. Thus, the recession only creates a modest skill mismatch. My model predicts that $\beta^e < 0$, so that the more severe skill mismatch in exposed zones affects unemployment through the job creation externality and the complementarity effect.

To interpret β^d and β^e as the direct and external effects of the skill mismatch, I require two key assumptions. First, that a workers' commuting zone of residence does not explain unobservable heterogeneity in skills or abilities within a skill group; workers in a given group are comparable across different commuting zones. Otherwise the measurement error in workers' skills could show up in β^e as a spurious external effect (see Acemoglu and Angrist, 2000).³¹ Second, that commuting zones that were highly exposed to the decline in routine-cognitive jobs do not differ in other characteristics that affect employment. Otherwise the effect of these characteristics could be misinterpreted as the external effect of skill mismatch.

With these assumptions in mind, I estimate equation (13) using as dependent variable the employment rate, the unemployment rate and the average log hourly wage for each skill group in each commuting zone. The data for each cell comes from the Census and the American Community Survey. To remove the role of sectoral shocks and differences in cyclicity across industries, I control for the 2000 employment share in manufacturing, construction, tradable and durable goods industries (all are measured using the 2000 Census) for each commuting zone and each skill group. In addition, in each commuting zone I control for factors that determine the labor supply and that I introduced above. I also include a full set of dummies for characteristics of each skill groups, including sex, age, region of residence and educational level. These controls guarantee that the estimate of β^d does not confound the direct effect of structural change with the effects of differences in education, female participation in the labor force, and life-cycle dynamics.

³¹To see why this is required, write an individual's true specialization in routine-cognitive jobs as $S_{jig} = GRC_g + \epsilon_{jig}$. Here, j indexes the individual, GRC_g is the observable group component that I observe and ϵ_{jig} is an unobservable individual component. Plugging this term instead of GRC_g in equation (13), shows that the regression estimate of β^e converges in probability to $\beta^e + \beta^d Cov(\epsilon_{jig}, RC_i|g)/Var(RC_i)$. Thus, this regression identifies the external effects if and only if $Cov(\epsilon_{jig}, RC_i|g) = 0$, which boils down to the stated assumption.

Panel A in Table 2 presents my estimates. In Columns 1 to 3, I focus on the average change in the employment rate, the unemployment rate, and the log of hourly wages from 2007 to 2009 and 2010—the recession years—as the dependent variables. The Column 1 estimates show that the direct effect was significant. From 2007 to 2009-2010, 10 percentage points of additional specialization in routine-cognitive jobs were associated with a 0.89 percentage point reduction in the employment rate of workers in that skill group (standard error=0.22 p.p.). As Column 2 confirms, the reduction in employment translated into a 0.64 percentage point increase in the unemployment rate of these workers (standard error=0.14 p.p.). Only about 20% of the displaced workers quit the labor force; the rest remained unemployed and searching for jobs.

These direct effects, however, do not account for to the full general equilibrium effect of structural change, which also encompasses the externalities created by the skill mismatch within local labor markets. From 2007 to 2009-2010, a 10 percentage point increase in the exposure to the decline in routine-cognitive jobs (roughly the gap between the least and the most exposed commuting zones) was associated with a 1.23 percentage point reduction in employment (standard error=0.3 p.p.). The joblessness translated into a 1.15 percentage point increase in the unemployment rate and a 2.67% reduction in wages (see Column 3). In line with the main prediction of my model, within local labor markets, the external effects of the skill mismatch significantly amplified unemployment and joblessness.

In the bottom rows of Panel A, I present two counterfactual escenarios to illustrate the economic importance of my estimates. From 2007 to 2009-2010, the unemployment rate increased by 3.1 percentage points. This increase would have been of 2.68 percentage points if all skill groups with above-average specialization in routine-cognitive jobs had experienced the same labor market outcomes as the average worker. In this scenario, the direct effect of structural change explains up to 14% of the observed increase in unemployment. Moreover, the increase in the unemployment rate would have been of 2.24 percentage points if, in addition, commuting zones with above-average exposure to routine-cognitive jobs had experienced the same labor market outcomes as the average zone. The external effects of the skill mismatch explain an additional 14% of the observed increase in unemployment. Although these are just a pair of many potential counterfactual scenarios, they show that both the direct and external effects of structural change had a sizable impact.

Columns 4 to 6 in Panel A show that the effects of structural change were long-lasting. In these models I focus on the average change in the employment rate, unemployment rate, and the log of hourly wages from 2007 to 2011, 2012 and 2013—the recovery years—as the dependent variables. The increase in joblessness observed during the crisis persisted during the recovery years, for both workers specialized in routine-cognitive jobs and in the commuting zones that were more exposed to structural change.

TABLE 2: Direct and external effects of mismatch on employment, unemployment and wages.

Dependent variable:	Recession years			Recovery years		
	Change from 2007 to 2009-2010			Change from 2007 to 2011-2013		
	Employment (1)	Unemployment (2)	Wages (3)	Employment (4)	Unemployment (5)	Wages (6)
<i>Panel A. Decomposition of direct and external effects.</i>						
Skill group's specialization in routine-cognitive jobs, GRC_g	-0.089*** (0.022)	0.063*** (0.014)	0.033 (0.030)	-0.099*** (0.023)	0.026** (0.011)	0.057 (0.034)
CZ's exposure to routine-cognitive jobs, RC_i	-0.107*** (0.034)	0.102*** (0.024)	-0.254*** (0.047)	-0.138*** (0.046)	0.106*** (0.031)	-0.421*** (0.075)
R squared	0.19	0.16	0.06	0.19	0.13	0.11
Observations	70534	70534	70232	106259	106259	105907
<i>Counterfactuals:</i>						
Mean dependent variable (p.p.)	-2.568	3.100	-1.681	-2.795	2.324	-6.659
Removing direct effect	-1.978	2.680	-1.899	-2.136	2.149	-7.035
Removing also external effect	-1.567	2.290	-0.924	-1.607	1.743	-5.420
<i>Panel B. Estimation of external effect controlling for skill group effects.</i>						
CZ's exposure to routine-cognitive jobs, RC_i	-0.105*** (0.034)	0.103*** (0.024)	-0.261*** (0.047)	-0.134*** (0.047)	0.108*** (0.030)	-0.427*** (0.075)
R squared	0.21	0.17	0.08	0.22	0.14	0.13
Observations	70534	70534	70232	106259	106259	105907
<i>Significance of skill groups:</i>						
<i>Panel C. Estimation of external effect controlling for observed heterogeneity.</i>						
CZ's specialization in routine-cognitive jobs, RC_i	-0.106*** (0.037)	0.111*** (0.025)	-0.221*** (0.048)	-0.114** (0.046)	0.105*** (0.030)	-0.398*** (0.068)
<i>Observed heterogeneity:</i>						
Cell specialization in analytic tasks	-0.021 (0.013)	0.011 (0.010)	-0.024 (0.023)	0.011 (0.012)	-0.009 (0.007)	-0.005 (0.029)
Cell specialization in routine-cognitive tasks	-0.024 (0.015)	0.013 (0.011)	0.086*** (0.020)	0.025 (0.015)	-0.015* (0.008)	0.106*** (0.038)
Cell specialization in routine-manual tasks	-0.014 (0.016)	-0.006 (0.011)	0.019 (0.026)	-0.018 (0.018)	-0.004 (0.009)	0.052* (0.031)
F-statistic for workers' skills	1.3	1.6	10.7	1.9	1.6	5.7
R squared	0.22	0.18	0.08	0.23	0.15	0.14
Observations	70531	70531	70229	106255	106255	105903
<i>Panel D. IV estimation of external effect controlling for skill group effects.</i>						
CZ's exposure to routine-cognitive jobs, RC_i	-0.179*** (0.040)	0.150*** (0.026)	-0.396*** (0.076)	-0.166*** (0.052)	0.106*** (0.035)	-0.595*** (0.113)
R squared	0.21	0.17	0.07	0.22	0.14	0.13
Observations	70534	70534	70232	106259	106259	105907
First-stage F statistic	224.4	239.0	224.5	224.6	224.6	224.6

Notes: Panel A presents estimates of the differential change in labor market outcomes from 2007 onward among commuting zones more exposed to structural change (the external effect), together with the differential changes for workers in skill groups directly exposed to structural change (the direct effect). The dependent variable is indicated in top of each column, as well as the period for which I estimate the model. Panel B presents estimates of the external effect of structural change controlling for a full set of skill-group dummies. Panel C presents estimates of the external effect of structural change controlling for a full set of skill-group dummies and observed cell heterogeneity. Panel D presents instrumental-variables estimates in which I instrument for the share of employment in routine-cognitive jobs in 2000 using the historical share in 1980. In all models, I allow the error term ε_{igt} to be correlated within States and over time, and within skill groups and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight each commuting zone \times skill group cell by its size in 2000.

To illustrate the effects of the decline in routine-cognitive jobs over time, Figure 16 plots year-by-year estimates of equation (13) (the estimates are computed relative to 2007, which

is the base year). The figure presents the average employment and unemployment rate in the sample from 2005 to 2013. I add the estimated differences for skill groups at the 90th percentile of exposure to structural change (in blue circles), together with their corresponding 90% confidence intervals. On top of these series, I also add the estimated external effects in commuting zones at the 90th percentile of exposure to structural change (in red triangles), together with their corresponding 90% confidence intervals. The series depicted by the red triangles corresponds to the predicted outcomes for workers in skill groups at the 90th percentile of specialization in routine-cognitive jobs who reside in commuting zones at the 90th percentile of exposure to structural change.

The figures show that, during the Great Recession, workers in skill groups that specialized in routine-cognitive jobs suffered from worst labor-market outcomes than other workers, even though the outcomes of the two groups moved in tandem before the great recession. Workers located in highly exposed commuting zones also suffered during the recession from worst labor-market outcomes; while differences across commuting zones were not causing any divergence before.

In the bottom panel of the figure, I present estimates using as dependent variable a measure of long-term unemployment, which I compute as the share of workers who currently are unemployed and report that they did not have a job during the last year. Because in my model the skill mismatch and its external effects depress the finding rate, the increase in unemployment should coincide with an increase in unemployment duration. In keeping with this logic, Figure 16 shows that the incidence of long-term unemployment drove the direct effect on workers specialized in routine-cognitive jobs, and the external effects of the skill mismatch in commuting zones exposed to this structural change. Contrary to models that emphasize how recurrent job losses or an increase in separations affect displaced workers (see Lillien, 1982; and Jarosch, 2014), my evidence shows that unemployment is driven by a decline in job-finding rates and turnover (as Figure 3 in the introduction also confirms), and is associated with an increase in long-term unemployment.³²

The distinctive prediction of my model is that the external effects of the skill mismatch are important. I devote the rest of this section to exploring in detail these external effects and their robustness.

A potential concern is that the external effects are picking up unobserved heterogeneity within skill groups, which violates the first assumption stated above. For instance, the index

³²In the Data Appendix, I analyze data from the Longitudinal Employer-Household Dynamics. Unfortunately, the public-use version of this data cannot be broken into finely defined skill groups and so it must be aggregated at the commuting zone level. I find that, during the Great Recession, highly exposed commuting zones experienced a decline in job-finding rates and a decline in turnover (separation rates declined slightly as well, especially during the recovery years).

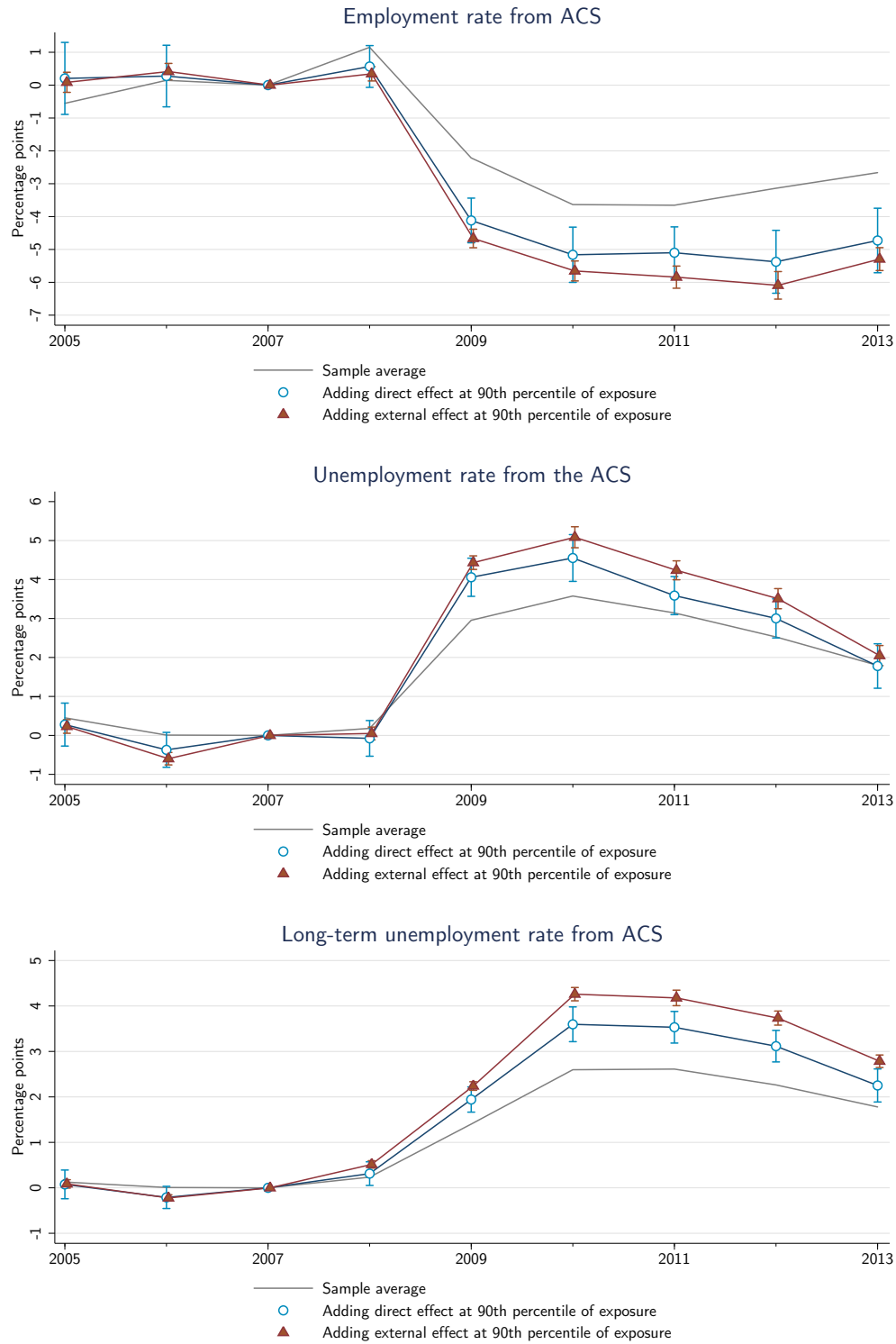


FIGURE 16: Estimates for the external effect in commuting zones at the 90th percentile of exposure to routine-cognitive jobs, and estimates for the direct effect on skill groups at the 90th percentile of specialization in routine-cognitive-jobs, relative to the sample average. Data from the American Community Survey.

GRC_g may not capture the full bundle of skills held by workers. Or in commuting zones where routine-cognitive jobs abound, workers within a skill group may have less experience

with analytical tasks.

I address this concern in several ways. First, Panel B in Table 2 reproduces the estimates of the top panel but includes a full set of skill-group dummies. Instead of measuring the direct effect using groups' specialization patterns, GRC_g , these models control flexibly for the fixed characteristics of skill groups (observable and not). These models exploit within-group variation and compare workers in the same skill group across different local labor markets, so they do not rely on my particular measure for GRC_g being accurate. I find that the external effects at the commuting-zone level remain roughly unchanged. The skill-group dummies are jointly significant in all models, as the large F statistics reported in the bottom row of the Panel indicates; during the Great Recession skill groups were an important determinant of labor market outcomes.

Second, to gauge the importance of unobserved heterogeneity within skill groups, Panel C in Table 2 reproduces the estimates of Panel B but also controls for the cell-level specialization in analytical and routine(cognitive or manual) tasks. I compute these measures of specialization as the year 2000 employment share in highly analytical or routine jobs for each skill group \times commuting zone cell. These measures capture observed differences within skill groups across commuting zones. Reassuringly, there is no evidence of any significant source of heterogeneity within skill groups that affects labor market outcomes (except for wages). In Columns 1, 2, 4 and 5, the point estimates for these observed differences are a fraction of the external effects' estimates, and are not jointly significant. As expected, my estimates of the external effects remain significant and of a similar size to the previous models.³³

In additional exercises not reported here, I obtained similar estimates of the external effects if, instead of using the skill groups described above, I partition workers by the occupation they report in the Census and American Community Survey. Though reported occupations may be informative about workers' skills and relevant labor-market experience, this exercise faces the problem that reported occupations are a bad control (see Angrist and Pischke, 2008).

A final concern is that the external effects are picking up unobservable differences across commuting zones. To address it, I present estimates in Panel D in which I instrument the external effects (using the specification in Panel B) with the 1980 share of employment in routine-cognitive jobs in each commuting zone. The first-stage F statistic of roughly 200 suggests there is a considerable degree of persistence in specialization patterns. The instrumental-

³³Moreover, this is a common assumption used in the literature that decomposes the increase in unemployment during the Great Recession in structural and other factors. For instance, Kroft et al. (2014) and Barnichon and Figura (2015) decompose the finding rate of workers in a component determined by demographic characteristics (as the ones I use here) and a component that they attribute to duration dependence. Both decompositions preclude the existence of unobserved heterogeneity, which is interpreted as pure duration dependence. In a different context, Beaudry, Green and Sand (2012) rely on a similar assumption to estimate the spillovers of the industrial composition of a local labor market.

variables estimates, which are slightly larger, support my previous findings. The historical persistence of specialization patterns provides a plausible exogenous source of identification, as it removes recent changes in exposed labor markets that could affect their outcomes during the Great Recession. In the next Subsection, I show that commuting zones that were highly exposed to routine-cognitive jobs historically did not fare badly during the 1990 recession. Since this recession preceded the decline in routine-cognitive jobs, this result lends support to the view that exposed commuting zones do not differ in historical and unobservable factors that shape their response to recessions.

Overall, I see the assumptions required to interpret my estimates of β^e as the external effects of the skill mismatch as plausible. There may be small differences within skill groups across commuting zones, but it is unlikely that these explain the large effects at the commuting-zone level that I estimate (see Panel C in Table 2). A more plausible interpretation of the large effects found at the commuting-zone level and that one would miss in the national-level comparison across skill groups, is that they reflect the external costs of adjustment borne out at the local labor market.

My model predicts that due to the job creation externality, these adjustment costs are borne out by workers who specialize in professional jobs, although they are not directly affected by the computerization of routine tasks. Moreover, The paucity of novel and stepping-stone jobs that emerges as a consequence of the skill mismatch also affects workers who specialize in routine jobs (both cognitive and manual). Because of the lack of employment and retraining opportunities in professional jobs, workers cannot redeploy to professional jobs easily and are forced to stay at service and routine jobs or remain unemployed. There should be no external effects on workers who specialize in managerial jobs—which do not absorb displaced workers—or service jobs—which absorb displaced workers but do not require additional skills or much training.

I test these predictions by estimating the incidence of the external effects on different groups of workers. I estimate the following extension of equation (13):

$$\Delta Y_{igt} = \sum_o \beta_o^e RC_i \times S_{go} + \alpha_{gt} + \Gamma_i X_i + \varepsilon_{igt}. \quad (14)$$

Here, S_{go} is the specialization of each skill group in different occupational categories, o , including managerial jobs, professional jobs, routine-cognitive and manual jobs and service jobs. I measure specialization patterns using the 2000 share of employment for each skill group in the different occupational categories listed above. Because each occupation falls within one of these categories, $\sum_o S_{go} = 1$ and the external effects β^e are a weighted average of the coefficients β_o^e , which decompose the incidence of spillovers on different groups of workers. I control for a full set of skill-group dummies allowed to vary by year, α_{gt} . Equation (14) identifies the incidence of the external effects of the skill mismatch by comparing workers in the same skill

group across different commuting zones. I pool the change in labor market outcomes from 2007 to all years from 2009 to 2013, so that I estimate the average incidence of external effects during the Great Recession and its recovery,

Table 3 presents my estimates. Columns 1 and 2 use the employment rate as the dependent variable; Columns 3 and 4 use the unemployment rate; and Columns 5 and 6 use the log of hourly wages. Consistent with the predictions of my model, the estimates in Columns 1 and 3 show that the external effects of the skill mismatch affect workers who specialize in professional jobs, routine-cognitive jobs and routine-manual jobs. A 10 percentage point increase in my measure of mismatch at the commuting zone level is associated with a large decline of 3.4 percentage points in the employment rate of workers who fully specialize in professional jobs and an increase of 0.8 percentage points in their unemployment rate. These large effects are comparable in size to the direct effects of structural change on workers who specialize in routine-cognitive jobs, which underscores the importance of external effects in the data.

As expected, I also estimate negative external effects on the employment of workers who specialized in routine-cognitive and routine-manual jobs. This last group comprises production workers who were also leaving manufacturing jobs during my period of analysis.

Remarkably, there is no evidence of spillovers on workers who specialized in managerial or service jobs, and for which my theory predicts there should be none. In line with these findings, in additional results that I do not report, I find that spillovers were concentrated among workers who specialized in occupations that require an intermediate level of training, such as professional jobs, but were not borne out by workers specialized in occupations that require the most training and the least training. These findings are reassuring because we would not expect displaced workers to become radiologists, who require 15 years of specialized training and several licenses. Nor should the deployment of routine-cognitive workers affect hiring in jobs that do not require much training or specialized skills.

Workers who specialized in routine-cognitive jobs and who resided in the most exposed commuting zones also suffered a decline in their wage of about 13.8%. Yet despite the negative external effect on their employment rate, workers in the most exposed commuting zones who had the skill set needed for professional jobs experienced a wage increase of 3.78%.

Although my model predicts that both wages should decline, this last finding is easy to reconcile with a slight variant of my theory. In my model, the behavior of wages is pinned down by the assumption that the number of vacancies adjusts immediately, which search models make for tractability. In the Theory Appendix I relax this assumption and assume that existing vacancies close gradually when they become unprofitable. In this case, the skill mismatch always increases the wage of skilled workers in the short run and reduces it in the medium run, when firms close unprofitable vacancies. Intuitively, when unskilled workers abound, the outside option of firms matched to a skilled worker deteriorate in the short

TABLE 3: Estimates of the incidence of external effects on different skill groups.

	Dependent variable:					
	Employment rate		Unemployment rate		log of wages	
	(1)	(2)	(3)	(4)	(5)	(6)
<i>External effect on workers specialized in:</i>						
	<i>Panel A. Incidence of external effects on workers specialized in different occupations.</i>					
Managerial jobs	0.051 (0.180)	0.045 (0.178)	0.084 (0.091)	0.092 (0.091)	0.250 (0.332)	0.262 (0.326)
Professional jobs	-0.343*** (0.092)	-0.340*** (0.090)	0.081* (0.048)	0.086* (0.046)	0.378** (0.179)	0.481*** (0.184)
Routine-cognitive jobs	-0.258** (0.116)	-0.259** (0.114)	0.245*** (0.071)	0.242*** (0.070)	-1.384*** (0.227)	-1.419*** (0.218)
Routine-manual jobs	-0.248** (0.095)	-0.248*** (0.093)	0.145*** (0.049)	0.150*** (0.049)	-0.735*** (0.157)	-0.690*** (0.154)
Services	0.182 (0.230)	0.173 (0.224)	-0.026 (0.120)	-0.012 (0.118)	0.306 (0.463)	0.343 (0.443)
R-squared	0.21	0.21	0.15	0.15	0.14	0.16
Observations	176793	176793	176793	176793	176139	176139
<i>External effect on workers specialized in:</i>						
	<i>Panel B. Incidence of external effects on workers specialized in tradable occupations.</i>					
Tradable industries	-0.133* (0.069)	-0.133** (0.066)	0.151*** (0.045)	0.153*** (0.044)	-0.075 (0.104)	-0.010 (0.102)
Non-tradable industries	-0.165 (0.125)	-0.172 (0.123)	0.096* (0.057)	0.103* (0.055)	-0.972*** (0.213)	-0.995*** (0.207)
R-squared	0.21	0.21	0.15	0.15	0.14	0.16
Observations	176793	176793	176793	176793	176139	176139
<i>Additional covariates:</i>						
Employment change by occupation		✓		✓		✓

Notes: The table presents estimates of the incidence of the external effect of structural change on different groups of workers, identified in each row and panel. The dependent variable is the indicated in the top rows. In each column I pool together the change in each dependent variable from 2007 to 2009, 2010, 2011, 2012 and 2013. All models include a full set of skill-group dummies allowed to vary by year. When estimating this equation, I allow the error term ε_{igt} to be correlated within States and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, following a common practice in the literature, I weight each commuting zone \times skill group cell by its size in 2000.

run. This opportunity-cost effect allows skilled workers to obtain higher wages (see Acemoglu 1997b). However, this effect reverts as firms exit the market and the outside option for skilled workers deteriorates, shifting the balance of the bargaining between skilled workers and firms.

The behavior of wages in the data weighs against alternative theories that emphasize nominal rigidities or contractual frictions as the main limit to the redeployment of displaced workers, at least when such redeployment requires retraining workers. Unlike models with wage rigidities, unemployment is associated with a *large* decline in average wages for routine-cognitive workers and an increase in wage dispersion. In fact, this observed wage decline roughly matches the behavior of wages in my numerical calibrations even though wages are not exogenously rigid in my model.

The incidence of external effects and the behavior of wages also weighs against other interpretations of my findings. The precise patterns of incidence that I estimate are empirically

difficult to reconcile with the possibility that external effects reflect unobserved heterogeneity across commuting zones or skill groups.

One alternative explanation able to generate amplification at the commuting zone level, is that labor may have a decreasing marginal value in jobs receiving displaced workers (professional and service jobs). In this case, reallocation could directly affect the employment opportunities for workers specialized in professional jobs. This explanation does not fit several features of the data. First, in Columns 2, 4 and 6, I control directly for the change in the log of employment in professional, managerial and service jobs, which captures the potential decline in the marginal value of these occupations. Because these are bad controls (people change occupations due to the structural change. See Angrist and Pishke, 2008), I instrument them using the log of employment in each occupation at the start of the recession. It is reassuring to see that my estimates do not change in their presence. Second, this mechanism cannot explain why there are no spillovers on workers specialized in the service sector, which would also be subject to decreasing returns, or why do wages for skilled workers increase.

Another alternative emphasizes the role played by local demand spillovers. For instance, workers might consume less when they are displaced from routine-cognitive jobs and learn that their skills became obsolete. This mechanism reduces local demand and affects employment for other workers (see Beaudry, Portier and Green, 2014; and Acemoglu et al., 2014). However, demand externalities would have the strongest effect on workers in the highly non-tradable service sector and would lower the wage of workers who specialized in professional jobs, which are not the case in the data. Moreover, in Panel B I address this concern by decomposing the spillovers on workers who specialized in tradable and non-tradable industries (following the classification by Acemoglu et al., 2014). I find evidence of negative spillovers on both groups, which suggests that the external effects were not driven by local-demand spillovers on workers who specialized in non-tradable industries.

As a last test of the external effects, I present evidence of the complementarity effect. In my model, the complementarity effect implies that, in highly exposed labor markets, firms will create few stepping-stone jobs to help displaced workers retrain and relaunch their careers in professional jobs. To test this implication, I estimate an extension of equation (11):

$$\Delta Y_{igt} = \beta GRC_g + \alpha GRC_g \times \overline{RC}_i + \delta_i + \Gamma_g X_g + \varepsilon_{igt}. \quad (15)$$

Here, ΔY_{igt} is the change in the share of workers from skill group g in commuting zone i employed in different occupational categories. I focus on the change from 2007 to 2013, so that workers have had enough time to redeploy. This equation investigates whether in the top half of commuting zones that were more exposed to structural change (measured by the dummy $\overline{RC}_i = 1$), displaced workers redeployed to professional jobs at a lower rate—captured by α . On the other hand, β captures reallocation patterns in the bottom half of commuting

zones that had less exposure. I partial out all commuting zone characteristics, δ_i , and include all the controls for the demographic groups used previously.

Table 4 presents my results. Column 1 shows that workers who specialized in routine-cognitive jobs redeployed to professional jobs at a lower rate in the top half of commuting zones that were more exposed to structural change. In these areas, workers redeployed to service jobs instead. Column 5 shows that in commuting zones that had low exposure, workers redeployed to jobs that had high-training requirements, but among workers in highly exposed commuting zones this occurred less frequently. The estimates for the interaction term α are all sizable relative to the main effect, β , on the top row. These findings support the key mechanism that slows down the speed of adjustment in my model. Workers in highly exposed areas were not willing to accept wage cuts in order to take stepping-stone jobs because they anticipated that professional jobs would remain scant in their commuting zone for a while. In response, firms created few of these jobs, which hampered the redeployment of routine-cognitive workers to professional jobs.

TABLE 4: Transitions of displaced workers to other occupations in average and in highly exposed commuting zones.

	Average change from 2007 to 2013 in reported employment in:				
	Service jobs	Routine jobs	Professional jobs	Managerial jobs	High-training jobs
	(1)	(2)	(3)	(4)	(5)
Group's specialization in routine-cognitive jobs	0.060* (0.033)	-0.163*** (0.036)	0.071*** (0.022)	0.032 (0.028)	0.078** (0.031)
Difference in highly exposed labor markets	0.023*** (0.009)	0.000 (0.011)	-0.013** (0.006)	-0.010 (0.007)	-0.020** (0.009)
R-squared	0.10	0.08	0.06	0.07	0.07
Observations	35440	35440	35440	35440	35440

Notes: The table presents estimates of the differential reallocation patterns for workers in skill groups more exposed to structural change, and compares them to the corresponding estimates for the same groups in commuting zones in the top half of exposure to structural change. The dependent variable is the change from 2007 to 2013 in the share of workers who report the occupational group indicated in each column. All columns include a full set of commuting zone dummies and additional skill-group controls. When estimating this equation, I allow the error term ε_{igt} to be correlated within skill groups and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, following a common practice in the literature, I weight each commuting zone \times skill group cell by its size in 2000.

3.3 Interactions between structural change and recessions

In this section I explore if the decline in routine-cognitive jobs interacted with the Great Recession—as predicted by my model—or both phenomena simply overlapped in time.

The timing of the effects provides the first evidence of an interaction. Figures 3 and 16 show that from 2005 to 2007, exposed commuting zones and groups of workers who specialized in routine-cognitive jobs did not experience divergent labor market outcomes. Both the direct and external effects of structural change concentrated during the Great Recession (from 2007

to 2010) and the recovery years (from 2011 to 2013). This timing suggests that the recession exacerbated the effects of structural change. The economic boom during the previous years, on the other hand, masked these effects.

To further explore the timing of the effects, I use data from the County Business Patterns, which provide employment counts by industry on a yearly basis for each commuting zone. This limitation of the data restricts me to estimating models at the commuting-zone level. I estimate equation (12) at the commuting-zone level with the change in employment per working age population as the dependent variable for each year from 1988 to 2013, relative to 2000. Because the data is only available at the commuting zone level, the coefficients on RC_i capture a mixture of the direct and external effects of the skill mismatch. To control for the strong secular decline in manufacturing during these longer period I focus on non-manufacturing employment.

Figure 17 plots the average employment rate outside manufacturing, relative to the year 2000, in gray. I also plot in blue the estimated change in the employment rate relative to the year 2000 in commuting zones at the 90th percentile of exposure to the decline in routine-cognitive jobs. The 1990, 2001 and 2007 recessions and their respective recovery years are shaded in gray. Figure 18 closely examines the behavior of employment during these three recessions relative to the year that preceded the start of each recession.

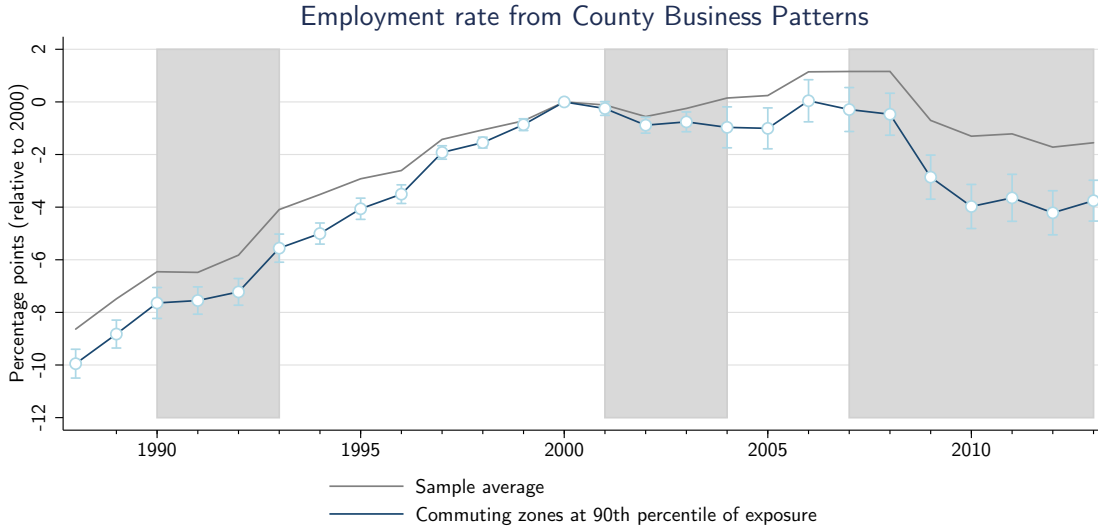


FIGURE 17: Estimated change in employment for commuting zones at the mean (in gray) and 90th percentile of exposure to structural change (in blue) relative to the year 2000. The light-blue bars plot 90% confidence intervals for the estimates in the highly exposed areas. Data from the County Business Patterns.

The figures reveal a clear pattern: the effects of the decline of routine-cognitive jobs concentrate during the 2001 and 2007 recessions. Exposed labor markets did not experience divergent employment paths before 2000 or during the booming years of 2004 to 2007. For

each additional 10 percentage points of exposure, employment from 2000 to 2004 declined by 2.87 percentage points (standard error=1.12). From 2007 to 2010, the corresponding decline in employment was 3.21 percentage points (standard error=0.41). From 2004 to 2007—years that had a strong economy and that fell between recessions—the corresponding decline in employment in exposed zones was only 0.51 percentage points (standard error=0.4).

For the 1990 recession I do not find any divergent path for employment in the most exposed commuting zones. In the figure that depicts the employment behavior around the 1990 recession, I measure the exposure of each commuting zone by the share of employment in routine-cognitive jobs in the 1990 Census, so that this measurement is predetermined. This serves as an useful placebo test because this recession preceded the decline in routine-cognitive jobs that started in the late 90s. The patterns that surround the 1990 recession confirm that highly exposed commuting zones only became more responsive to the business cycle when the decline of routine-cognitive jobs started.

In addition to the timing of the effects uncovered above, I also exploit cross-sectional variation in the incidence of the Great Recession to test directly for an interaction with structural change. I explore whether the decline of routine-cognitive jobs had a larger effect in commuting zones that, because of factors orthogonal to this structural change, suffered during the Great Recession from a sharp reduction in economic activity. To do this I estimate the model

$$\Delta Y_{it} = \beta RC_i + \theta_N DS_i + \alpha RC_i \times DS_i + \delta_t + \Gamma X_i + \varepsilon_{it}, \quad (16)$$

with DS_i a proxy for the decline in local economic activity at commuting zone i during the Great Recession. I explore two different proxies for DS_i : the loss in household net worth during the recession (see Mian and Sufi, 2014), and the increase in household leverage from 2000 to 2006 (see Mian, Rao and Sufi, 2013). Both measures explain the local decline in economic activity during the crisis.³⁴

In equation (16), β captures the local effects of the skill mismatch and θ the direct effect of a decline in demand on economic activity. The coefficient of the interaction term α captures the interaction between the decline in economic activity and structural change.

Table 5 reports estimates of equation 16. In Columns 1 to 4 the decline in net worth is the proxy for the decline in local economic activity. This measure is available for 365 commuting zones, which comprise my sample. Each panel presents results for a different dependent variable. Column 1 in Panel B shows that during the onset of the Great Recession,

³⁴These measures are available for a subset of counties that I aggregate to the commuting-zone level. Overall, I find that both DS_i and RC_i are only weakly correlated across commuting zones. Although not reported to save space, I also include quadratic terms for DS_i and RC_i , which guarantee that I am not capturing non linearities.

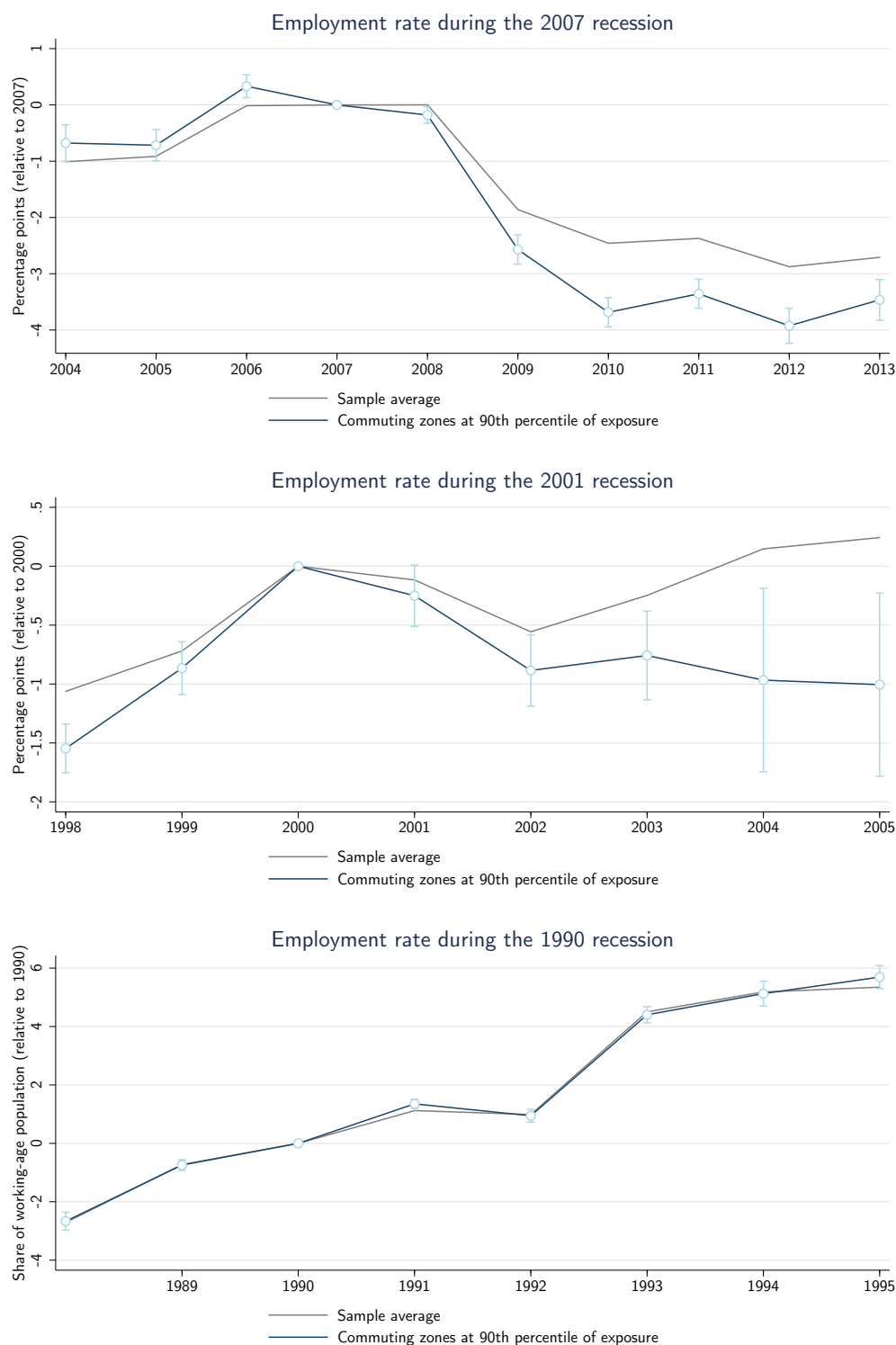


FIGURE 18: Estimated change in employment for commuting zones at the mean (in gray) and 90th percentile (in blue) of exposure to structural change. The light-blue bars plot 95% confidence intervals for a test of both series being different. Data from the County Business Patterns.

non-manufacturing employment decreased by 0.5 percentage points (standard error=0.19)

for every additional reduction of 10 percentage points in household net worth.³⁵ Panel C shows that the decline in employment caused a corresponding increase in unemployment of 0.6 percentage points. Although my sample contains only half the commuting zones in the data, I find that commuting zones that were exposed to the decline in routine-cognitive jobs saw a significant decline in employment and an increase in unemployment.

TABLE 5: Interaction between the local decline in aggregate demand and exposure to structural change in commuting zones.

Proxy for local decline in demand:	Decline in net worth during crisis				Increase in leverage before crisis			
	Change from 2007 to 2010		Change from 2007 to 2013		Change from 2007 to 2010		Change from 2007 to 2013	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>Panel A. Dependent variable: share of employment in routine-cognitive jobs.</i>								
Local economy decline	0.011 (0.012)	0.006 (0.015)	0.018 (0.013)	0.037* (0.019)	0.010** (0.005)	0.006 (0.004)	0.008** (0.004)	0.008 (0.005)
Share of routine jobs in non-manufacture	-0.129** (0.059)	-0.112*** (0.037)	-0.264*** (0.059)	-0.283*** (0.053)	-0.141*** (0.050)	-0.119*** (0.042)	-0.236*** (0.054)	-0.276*** (0.051)
Routine non-manufacture × Local economy decline		-0.010** (0.004)		-0.006* (0.003)		-0.002 (0.002)		-0.002 (0.001)
R-squared	0.15	0.17	0.23	0.25	0.12	0.13	0.21	0.22
Observations	365	365	365	365	606	606	606	606
<i>Panel B. Dependent variable: non-manufacturing employment.</i>								
Local economy decline	-0.050** (0.019)	-0.084*** (0.024)	-0.016 (0.024)	-0.051 (0.032)	-0.016*** (0.005)	-0.023*** (0.006)	-0.010 (0.007)	-0.020** (0.008)
Share of routine jobs in non-manufacture	-0.294*** (0.056)	-0.288*** (0.053)	-0.167** (0.072)	-0.094 (0.079)	-0.293*** (0.041)	-0.292*** (0.045)	-0.166*** (0.058)	-0.115* (0.067)
Routine non-manufacture × Local economy decline		-0.005** (0.002)		-0.008** (0.004)		-0.001 (0.001)		-0.002** (0.001)
R-squared	0.69	0.70	0.56	0.58	0.67	0.67	0.48	0.49
Observations	365	365	365	365	606	606	606	606
<i>Panel C. Dependent variable: unemployment rate.</i>								
Local economy decline	0.060*** (0.013)	0.085*** (0.015)	0.034*** (0.013)	0.039** (0.017)	0.018*** (0.004)	0.022*** (0.005)	0.011*** (0.004)	0.011*** (0.004)
Share of routine jobs in non-manufacture	0.093** (0.035)	0.068* (0.036)	0.047 (0.034)	0.018 (0.036)	0.090*** (0.026)	0.075** (0.034)	0.049 (0.030)	0.037 (0.035)
Routine non-manufacture × Local economy decline		0.004** (0.002)		0.004** (0.002)		0.001 (0.001)		0.001** (0.001)
R-squared	0.64	0.65	0.58	0.59	0.58	0.58	0.54	0.54
Observations	365	365	365	365	606	606	606	606

Notes: The table presents estimates of both the differential change in commuting zones that were more exposed to structural change and their interaction with the local decline in economic activity during the Great Recession and the recovery (as indicated in the top rows). The dependent variable is the change in the share of workers who were employed in routine-cognitive jobs (Panel A), the change in the non-manufacturing employment rate (Panel B) and the unemployment rate (Panel C). In Columns 1 to 4 the decline in the net worth is a proxy for the decline in local economic activity, while in Columns 5 to 8 the increase in leverage from 2002-2006 is the proxy. When estimating this equation, I allow the error term ε_{it} to be correlated within States and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, employing a common practice in the literature, I weight commuting zones by the size of their workforce in 2000.

In Panel A the dependent variable is the change in the share of employment in routine-cognitive jobs. These models confirm that, in highly exposed areas, the local decline in

³⁵My estimates are smaller than Mian and Sufi's estimate of 1.9 percentage points, which they obtained at the county level. Presumably, the differences result from my level of aggregation and the fact that I focus on all non-manufacturing employment while they concentrate in employment in service jobs. Here, I focus on non-manufacturing employment in order to exclude the highly tradable manufacturing sector, which should not be affected by the decline in local demand that DS_i measures.

economic activity caused a larger decline in the share of workers employed in routine-cognitive jobs. Column 2 shows this was the case from 2007 to 2010, and Column 4 shows this was the case from 2007 to 2013. My interpretation of this finding is that in the commuting zones that experience the largest declines in economic activity, firms restructured and stop hiring workers for routine-cognitive jobs at a fast rate. This prompted workers displaced from routine-cognitive jobs to search for other jobs and exacerbated the skill mismatch during the crisis and the subsequent recovery.

In line with this interpretation, the estimates for α in Column 2 of Panels B and C show that local demand shocks *amplified* the effects of structural change during the onset of the Great Recession. Column 4 shows that the interaction *propagated* demand shocks over time, so that their effects lasted until the recovery. The interactions are quantitatively relevant. In the average commuting zone, a 10 percentage point loss in net worth causes a .5 percentage point decrease in employment. By 2013, the effect would have already vanished, suggesting that on average the effects of demand shocks are short lived. In contrast, in a commuting zone that is at the 90th percentile of exposure, a 10 percentage point loss in net worth causes a .75 percentage point decrease in employment. By 2013, the employment rate would still be down by .56 percentage points, which suggests that in markets affected by structural change the effects of demand shocks are large and long lived. Panel C presents results for unemployment, which yield similar findings.

I obtain similar results when I use the increase in leverage from 2000 to 2006 as a proxy for the decline in economic activity in any given commuting zone during the Great Recession. In the results, reported in Columns 5 to 8, I have a bigger sample of 606 commuting zones. The interaction coefficients are more sizable but less precise than before.

4 CONCLUDING REMARKS

This paper argues that economies fail to adjust properly when they are affected by a severe skill mismatch. Plausible matching frictions that limit the ability of firms to direct their search efforts can have large aggregate effects when the mismatch is severe. Using U.S. data and through a study of the decline of routine-cognitive jobs, I find support for the aggregate implications of these externalities, which operate at the commuting-zone level.

Under the lens of my model, the external effects of structural change identified at the commuting-zone level constitute an externality, and this opens room for a wide range of temporary policies during periods of skill mismatch, especially during recessions, when the effects of structural change are likely to concentrate. Though unemployment is structural, policies aimed at increasing demand or government expenditure during the initial stages of a recession can increase welfare by raising the returns to skill upgrading during the recovery and

by avoiding the fast closure of old jobs during the crisis. Subsidizing training temporary can ease the adjustment of the economy, increase welfare and reduce some of the unemployment observed during the current recovery.

Behind the job creation externality is a key assumption: firms cannot perfectly direct their search efforts. If this was not the case, the search behavior of unskilled workers would not affect matching opportunities for skilled workers, which seems like a restrictive requirement. I believe that the random matching assumption is plausible, but whether this assumption holds remains an empirical question. My data supports the aggregate implications of this mechanism, but to evaluate this more fully additional micro-evidence is needed.

I plan to complement my evidence for local labor markets with a micro approach that is designed to understand how firms change their hiring patterns when they face a skill mismatch, whether their response varies during recessions, and whether it is consistent with the predictions of models of random matching. An extension of my model that allows firms to direct their search efforts based on multiple signals suggests that when a skill mismatch occurs, firms focus most of their recruiting efforts on candidates who project the best (but scarce) signals. At the same time they devote few resources to hiring and training new candidates who have little experience or qualifications. This shift in hiring practices can affect the rate at which workers find jobs and firms fill their vacancies. In future research I will study these issues in more detail through analyses of proprietary data on job openings, which include detailed job characteristics, requirements and a description of the tasks performed by employees.

In recent years some of these phenomena have affected routine-cognitive jobs. These jobs, which formerly demanded few requirements, are professionalizing, and firms now routinely ask for additional requirements and credentials. The increase in requirements partly reflects changes in supply (see Modestino, Shoag and Ballance, 2015) and a shift away from routine-cognitive tasks. That said, a report by Burning Glass Technologies indicates that although in recent years 65% of the current job openings for executive secretaries call for a bachelor's degree, only 19% of those currently employed in these jobs satisfy this requirement. The report also argues that, in some cases, the formal requirements do not correspond to observed changes in tasks performed by workers. For some firms the requirements might serve as restrictive recruiting filters that exclude suitable matches and reduce the rate at which firms fill vacancies. My model views these hiring practices unfavorably, seeing them as inefficient bottlenecks for the reallocation of displaced workers and the creation and discovery of new talent.

At the macro level, my model and empirical evidence suggest that there is value to a perspective that regards recessions as times of adjustment and reorganization. Once we adopt this view, the usual distinction between structural and business-cycle phenomena blurs and the two become intricately related. The interaction between structural factors and business

cycles can be a useful addition to models of search unemployment. My evidence suggests that the interactions can be quantitatively significant and go a long way towards explaining the propagation of otherwise short-lived demand shocks. The possibility of an interaction raises questions about policy and crisis management during periods of structural change as well as the timing of adjustments. I intend to address these questions in future work.

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A1 THEORY APPENDIX

The Theory Appendix has the following structure. First, I describe the details of the behavior for the state variables and the Bellman equations. Second, I discuss some special conditions that guarantee the existence and uniqueness of the equilibrium for the general case in which a, λ take any positive values. In this subsection I also characterize the asymptotic behavior of the economy and provide general lemmas that I will use throughout the appendix. Third, I provide the details of the limit in which $a, \lambda \rightarrow \infty$, and provide conditions for the uniqueness of an equilibrium. In this subsection I also provide the proof of Proposition 2, and comparative statics results for the effects of z, x, I, Ω on the finding rates and reservation wages. Fourth, I provide the details of the proof for Proposition 3, 4 and 5. Finally, I derive the constraint efficient allocation in the general case in which a, λ take any positive values.

A1.1 Derivation of state variables and Bellman equations:

Derivation of the state variables behavior. The state variables of the model include the share of skilled workers x , the number of skilled unemployment workers u_s , the number of skilled unemployment workers u_u , and the number of workers of each type employed in different jobs e_j^k , where k indexes the task performed and j the type of worker.

The behavior of unemployment for both groups is given by:

$$\begin{aligned}\dot{u}_s &= \lambda x - (\Delta f(\theta_s^n) + (1 - \Delta)f(\theta_s^r))u_s + \delta u_u \\ \dot{u}_u &= \lambda(1 - x) - \left(\frac{\Delta}{1 + I} \pi f(\theta_s^n) + \frac{\Delta}{1 + I} (1 - \pi) f(\theta_u^l) + \frac{(1 - \Delta)}{1 + I} f(\theta_u^r) + \frac{I}{1 + I} f(\theta_u^o) \right) u_u - \delta u_u\end{aligned}$$

Using these expressions, I can calculate the behavior of the unemployment rate, u , and the share of skilled workers among the unemployed as:

$$\begin{aligned}\dot{u} &= \lambda(1 - u) - u\gamma f_s - u(1 - \gamma)f_u, \\ \dot{\gamma} &= (1 - \gamma)\gamma(f_u - f_s) + \lambda \frac{x - \gamma}{u} + (1 - \gamma)\delta,\end{aligned}$$

with f_s and f_u defined in equation (3) in the main text. To save on notation, these expressions assume all matches produce a positive surplus and are always formed. In the general case, when the surplus of a job is negative or zero, firms and workers reject these matches.

The behavior employment counts in each job is given by

$$\begin{aligned}\dot{e}_s^n &= \Delta f(\theta_s^n) u \gamma - \lambda e_s^n & \dot{e}_u^n &= \frac{\Delta}{1 + I} \pi f(\theta_s^n) u (1 - \gamma) - \lambda e_u^n \\ \dot{e}_u^l &= \frac{\Delta}{1 + I} (1 - \pi) f(\theta_u^l) u (1 - \gamma) - (\lambda + \alpha) e_u^l & \dot{e}_s^r &= (1 - \Delta) f(\theta_s^r) u \gamma - \lambda e_s^r \\ \dot{e}_u^r &= \frac{1 - \Delta}{1 + I} f(\theta_u^r) u (1 - \gamma) - \lambda e_u^r & \dot{e}_u^o &= \frac{I}{1 + I} f(\theta_u^o) u (1 - \gamma) - \lambda e_u^o.\end{aligned}\tag{A1}$$

Derivation of the state variables behavior. I now derive the Bellman equation for the surplus of a match. Denote by J_j^k the firm surplus and E_j^k the worker surplus. These surpluses are given by the Bellman equations:

$$rJ_j^k - \dot{J}_j^k = z_j^k - w_j^k - \lambda J_j^k \quad rE_j^k - \dot{E}_j^k = w_j^k + \lambda(U_j - E_j^k). \quad (\text{A2})$$

Here, z_j^k is the flow value of the match production (equal to $z(t)$ and adjusted by workers' productivity if needed). Also $-\lambda J_j^k$ and $\lambda(U_j - E_j^k)$ are the losses incurred by the firm and worker in the event that the match separates (recall that the firm outside option is set to zero by free entry).

Nash bargaining implies that $\beta J_j^k = (1 - \beta)(E_j^k - U_j)$, and $\beta \dot{J}_j^k = (1 - \beta)(\dot{E}_j^k - \dot{U}_j)$. Multiplying the first expression in equation (A2) by β , the second by $(1 - \beta)$, and subtracting them yields the following formula for the wage rate:

$$w_j^k = \beta z_j^k + (1 - \beta)(rU_j - \dot{U}_j).$$

Plugging the wage in the equation for \dot{J}_j^k yields:

$$(r + \lambda)J_j^k - \dot{J}_j^k = (1 - \beta) \left[z_j^k - (rU_j - \dot{U}_j) \right].$$

Nash bargaining implies that $J_j^k = (1 - \beta)S_j^k$. Therefore, we obtain:

$$(r + \lambda)S_j^k - \dot{S}_j^k = z_j^k - (rU_j - \dot{U}_j),$$

which is the expression used in the main text.

In the case of stepping-stone jobs, the derivation is different since I have to take into account the gains from training. In this case:

$$\begin{aligned} rJ_u^l - \dot{J}_u^l &= z_u^l - w_u^l - \lambda J_u^l + \alpha(J_s^n - J_u^l) \\ rE_u^l - \dot{E}_u^l &= w_u^l + \lambda(U_j - E_u^l) + \alpha(E_s^n - E_u^l). \end{aligned} \quad (\text{A3})$$

Here, z_u^l is the flow value of the match production, $z(t) - q^n - q^l$, adjusted by workers' productivity and training costs. $-\lambda J_u^l$ and $\lambda(U_j - E_u^l)$ are the losses on the firm and worker, respectively, in the event that the match is exogenously separated. $\alpha(J_s^n - J_u^l)$ and $\alpha(E_s^n - E_u^l)$ are the gains on the firm and worker, respectively, in the event that the worker becomes skilled.

Nash bargaining implies that $\beta J_u^l = (1 - \beta)(E_u^l - U_u)$, and $\beta \dot{J}_u^l = (1 - \beta)(\dot{E}_u^l - \dot{U}_u)$. Multiplying the first expression in equation (A3) by β , the second by $(1 - \beta)$, and subtracting them yields the wage:

$$w_u^l = \beta z_u^l + (1 - \beta)(rU_j - \dot{U}_j) - (1 - \beta)\alpha(U_s - U_u)$$

This equation shows that, as mentioned in the text, workers willingness to acquire skills reflects in lower wages at stepping-stone jobs (see Becker, 1964).

Plugging the wage in the equation for \dot{J}_u^l yields:

$$(r + \lambda)J_u^l - \dot{J}_u^l = (1 - \beta) \left[z_u^l - (rU_j - \dot{U}_j) + \alpha(U_s - U_l) \right] + \alpha(J_s^n - J_u^l).$$

Nash bargaining implies that $J_u^l = (1 - \beta)S_u^l$. Therefore, we obtain:

$$(r + \lambda)S_u^l - \dot{S}_u^l = z_u^l - (rU_j - \dot{U}_j) + \alpha(S_s^n - S_u^l) + \alpha(U_s - U_l).$$

The expression presented in the main text incorporates the slight variation that the worker and the firm have the option value of not incurring in training costs if it is not profitable:

$$(r + \lambda)S_u^l = z(t) - q^n + \max\{-q^l + \alpha(U_s - U_u) + \alpha(S_s^n - S_u^l), 0\} - (rU_u - \dot{U}_u) + \dot{S}_u^l.$$

Minimal set of state variables required to compute equilibrium. In the main text, I define the equilibrium based on fewer state variables than the set introduced above. The reason for doing that is that the equilibrium admits a recursive structure in which surpluses and γ —the share of skilled workers among the unemployed—determine tightness, tightness determines workers reservation wages and this feeds back into the surplus. Unlike the traditional search model, the fact that γ affects tightness—see equation (8) in the main text—implies that to determine the path for surplus and tightness we have to keep track of their joint behavior with γ .

To characterize the behavior of γ , I need to keep track of x, u, I and s . As equation (2) shows, the behavior of these variables only depend on tightness and their current values, so I can determine the equilibrium by focusing on this subset of the state variables. This is the minimal set of state variables required to characterize labor market tightness.

A1.2 Properties of the equilibrium and steady state behavior

Steady-state behavior of the economy.

PROOF OF PROPOSITION 1. The equation for \dot{x} shows that x converges monotonically to 1. Thus, in any steady state we have $x(t) \rightarrow 1$. Moreover, the exogenous behavior of $I(t)$ implies $I(t) \rightarrow 0$ by assumption.

For $x(t) = 1$ and $I(t) = 0$, the equilibrium conditions for the steady state are given by:

$$\begin{aligned} S_s^{n*} &= \frac{z - w_s^*}{r + \lambda}, & S_s^{r*} &= \frac{z - w_s^*}{r + \lambda}, \\ S_u^{n*} &= \frac{z - q^n - w_u^*}{r + \lambda}, & S_u^{r*} &= \frac{z - w_u^*}{r + \lambda}, \\ S_u^{o*} &= \frac{z - w_u^*}{r + \lambda}, & S_u^{l*} &= \frac{z - q^n - w_u^* + \max\{-q^l + \alpha\Omega^*\}}{r + \lambda}, \end{aligned}$$

with w_s^*, w_u^* the reservation wage of skilled and unskilled workers respectively, and $\Omega^* = U_s^* - U_u^*$ the incentives to acquire skills. Moreover, the reservation wages satisfy:

$$\begin{aligned} w_s^* &= rU_s^* = b + \Delta\beta f(\theta_s^{n*}) \max\{S_s^{n*}, 0\} + (1 - \Delta)\beta f(\theta_s^{r*}) \max\{S_s^{r*}, 0\}, \\ w_u^* &= rU_u^* = b + \Delta\beta[\pi f(\theta_s^{n*}) \max\{S_u^{n*}, 0\} + (1 - \pi)f(\theta_u^{l*})] \max\{S_u^{l*}, 0\} \\ &\quad + (1 - \Delta)\beta f(\theta_u^{r*}) \max\{S_u^{r*}, 0\} + \delta\Omega^*. \end{aligned}$$

These formulas imply that $S_s^{n*} = S_s^{r*} = S_s^*$. Moreover, since $x(t) = 1$, we have that in steady state $\gamma^n = 1$ and $\theta_s^{n*} = \theta_s^{r*} = \theta_s^*$. The equilibrium surplus S_s^* and θ_s^* are therefore equal to what one would obtain in the traditional search and matching model with homogeneous workers and jobs, and given by:

$$(1 - \beta)(1 - b) = \frac{r + \lambda + \beta\theta^* q(\theta^*)}{q(\theta^*)} \kappa \quad u = \frac{\lambda}{\lambda + f(\theta^*)}.$$

Although there are no unskilled workers in steady state, we can compute θ_u^{l*} and θ_u^{r*} for completeness. We have that $S_u^{l*} = S_u^l(w_u^*)$, $\theta_u^{l*} = \theta_u^l(w_u^*)$ are implicit and decreasing functions of w_u^* . The same holds for $S_u^{r*} = S_u^r(w_u^*)$, $\theta_u^{r*} = \theta_u^r(w_u^*)$ and $S_u^{n*} = S_u^n(w_u^*)$. Plugging these expressions in the equation for w_u^* we obtain

$$w_u^* = rU_u^* = b + \Delta\beta[\pi f(\theta_s^*) \max\{S_u^n(w_u^*), 0\} + (1 - \pi)f(\theta_u^l(w_u^*))] \max\{S_u^l(w_u^*), 0\}.$$

Since the left-hand side is increasing in w_u^* while the right hand side decreases, this equation defines a unique steady-state value for w_u^* . This reservation wage determines the steady-state tightness and surplus for jobs employing unskilled workers. ■

Dynamic equilibrium in the general case in which a, λ take any value.

PROPOSITION A1 *There exists a threshold $\bar{\beta} \in [0, 1]$ such that, for $\beta \leq \bar{\beta}$, the equilibrium exists and is unique.*

PROOF. Consider the limit case in which $\beta \rightarrow 0$. In this case, we have that $w_s, w_u \rightarrow b$.

Since w_s, w_u are pinned down by the value of leisure, the surpluses of different jobs and the tightness become jump variables, which do not depend on the path of future wages and only depend on the current value of $z(t)$.

In particular, the surpluses are given by

$$\begin{aligned} S_s^n(t) &= \frac{z(t) - b}{r + \lambda}, & S_s^r(t) &= \frac{z(t) - b}{r + \lambda}, \\ S_u^n(t) &= \frac{z(t) - q^n - b}{r + \lambda}, & S_u^r(t) &= \frac{z(t) - b}{r + \lambda}, \\ S_u^o(t) &= \frac{z(t) - b}{r + \lambda}, & S_u^l(t) &= \frac{z(t) - q^n - b}{r + \lambda}, \end{aligned}$$

And the tightness for different jobs is given by the free entry-conditions, which only depend on $z(t)$ and $\gamma(t)$.

That this is the unique solution for surpluses and tightness follows from the same argument presented in Pissarides (1985). Other values imply an explosive behavior for tightness and surpluses. Thus, there is a unique equilibrium path for finding rates, tightness, value functions and reservation wages. Since all equations and maps determining surpluses and tightness are continuous in β , these results extend to $\beta \in [0, \bar{\beta}]$.

The tightness at different jobs and the exogenous decline in $I(t)$ imply a unique and deterministic path for the finding rates, $f_s(t)$ and $f_u(t)$. Thus, the steady state behavior is pinned down by the solution to the boundary problem:

$$\begin{aligned}\dot{x} &= \delta u(1 - \gamma(t)), \\ \dot{u} &= \lambda(1 - u(t)) - u(t)\gamma(t)f_s(t) - u(t)(1 - \gamma(t))f_u(t), \\ \dot{\gamma} &= (1 - \gamma(t))\gamma(t)(f_u(t) - f_s(t)) + \lambda \frac{x(t) - \gamma(t)}{u(t)} + (1 - \gamma(t))\delta,\end{aligned}$$

coupled with an initial condition for $x(0), u(0), \gamma(0)$. ■

Remark: Though the proof of the first numeral relies on the limit case $\beta \rightarrow 0$, and does not provide any intuition of what values of β lead to a unique equilibrium, the following subsection provides a tighter characterization of this threshold in the empirical relevant case in which gross flows are large.

Remark 2: The key simplification in the proposition is that w_s, w_u do not change over time and converge rapidly to their steady state values. In the general case, the change over time of both reservation wages introduces additional complications and in some cases multiplicities.

Additional properties of the transition in the general case.

LEMMA A1 *In any equilibrium, we have $S_s^n > S_u^n$ at all points in time.*

PROOF. I first prove this is the case in steady state. Suppose by way of contradiction that $S_s^{n*} \leq S_u^{n*}$. Since $S_u^{l*} \geq S_u^{n*}$ and $S_u^{r*} \geq S_u^{n*}$, we have that $S_u^{l*}, S_u^{r*}, S_u^{n*} \geq S_s^{n*} = S_s^{r*}$.

Since unskilled workers would produce a higher surplus at all jobs, we have $w_u^* \geq w_s^*$. But then $S_s^{n*} = \frac{z - w_s^*}{r + \lambda} > \frac{z - q^n - w_u^*}{r + \lambda} = S_u^{n*}$, which yields a contradiction.

Now, I prove the same holds along the transition. In particular, I prove that if $S_s^n(T) \leq S_u^n(T)$ for some T , we have $S_s^n(t) < S_u^n(t)$ for all $t > T$. However, this contradicts the fact that in steady state we have $S_s^{n*} > S_u^{n*}$.

Suppose $S_s^n(T) \leq S_u^n(T)$. We have that $S_u^l(T) \geq S_u^n(T)$ since stepping-stone jobs have the additional option value of actually training workers. We also have that $S_u^r(T) = S_u^o(T) > S_u^n(T)$, since the opportunity cost of workers is the same in all these jobs, but unskilled

workers are more productive at regular and old jobs. Therefore, if $S_s^n(T) \leq S_u^n(T)$, we have $S_u^l(T), S_u^r(T), S_u^n(T), S_u^o(T) \geq S_s^n(T) = S_s^r(T)$.

The equation for $w_s(T)$ and $w_u(T)$ imply that $w_u(T) \geq w_s(T)$. Plugging this inequality in the Bellman equation for $S_s^n(t)$ and $S_u^n(t)$ at time T shows that the inequality $S_u^n(T) \geq S_s^n(T)$ can only hold if $\dot{S}_u^n(T) > \dot{S}_s^n(T)$. However, this implies that $S_s^n(t)$ appreciates more than $S_s^n(t)$ over time, and since $S_u^n(T) \geq S_s^n(T)$, we have $S_s^n(t) < S_u^n(t)$ for all $t > T$, which yields the desired contradiction. ■

A1.3 Details of the limit when $a, \lambda \rightarrow \infty$.

Formal description of the limit. I start by characterizing the equilibrium conditions in this limit case.

Define $\tilde{f}_s = f_s/\xi$, $\tilde{f}_u = f_u/\xi$ and $\tilde{f}(\theta) = f(\theta)/\xi$, $\tilde{q}(\theta) = q(\theta)/\xi$.

Taking the limit $\xi \rightarrow \infty$, we obtain that the behavior of the state variables converges to:

$$\begin{aligned} \dot{x} &= \alpha s + \delta u(1 - \gamma), & 0 &= \tilde{\lambda}(1 - u) - u\gamma\tilde{f}_s - u(1 - \gamma)\tilde{f}_u, \\ 0 &= u(1 - \gamma)\frac{\Delta}{1 + I}(1 - \pi)\tilde{f}(\theta_u^l) - \tilde{\lambda}s, & 0 &= (1 - \gamma)\gamma(\tilde{f}_u - \tilde{f}_s) + \tilde{\lambda}\frac{x - \gamma}{u}, \\ \dot{I} &= -Iv(t). \end{aligned} \tag{A4}$$

This follows by noting that the right-hand side of the equations for $\dot{u}, \dot{\gamma}$ and \dot{s} explode otherwise. A complementary intuition for this limit is that these stock variables converge to the stocks determined by current finding and separation rates at a rate of the order of $(a + \lambda)$ over time.

Rearranging the equations in (A4) yields the system in equation (9) that determines the behavior of γ, s, u in terms of the finding rates and the share of skilled workers in the economy.

To compute the equilibrium tightness, define the normalized surplus as $\lim_{\xi \rightarrow \infty} \xi S_j^k = \tilde{S}$. The normalized surpluses are given by

$$\begin{aligned} \tilde{S}_j^k(t) &= \lim_{\xi \rightarrow \infty} \xi \int_t^\infty e^{-(r + \tilde{\lambda}\xi)(\tau - t)} h_j^k(\tau) d\tau \\ &= \lim_{\xi \rightarrow \infty} \frac{\xi}{r + \tilde{\lambda}\xi} h_j^k(t) + \frac{\xi}{r + \tilde{\lambda}\xi} \int_t^\infty e^{-(r + \tilde{\lambda}\xi)(\tau - t)} \frac{dh_j^k}{dt}(\tau) d\tau \\ &= \frac{h_j^k(t)}{\tilde{\lambda}}. \end{aligned}$$

with $h_j^k(t)$ given by the right-hand side of the flow value of a match. The second line uses integration by parts. This line does not require $h_j^k(t)$ to be differentiable, but simply to have a representation of the form $h_j^k(t) = \int_0^t \frac{dh_j^k}{dt}(\tau) d\tau$ for some integrable function $\frac{dh_j^k}{dt}$.

Thus, normalized surpluses are well defined (without the normalization, the surplus converges to zero) and given by the solution to the static system:

$$\begin{aligned}\tilde{S}_s^n &= \frac{z(t) - w_s}{\tilde{\lambda}}, & \tilde{S}_s^r &= \frac{z(t) - w_s}{\tilde{\lambda}}, \\ \tilde{S}_u^n &= \frac{z(t) - q^n - w_u}{\tilde{\lambda}}, & \tilde{S}_u^r &= \frac{z(t) - w_u}{\tilde{\lambda}}, \\ \tilde{S}_u^o &= \frac{z(t) - w_u}{\tilde{\lambda}}, & \tilde{S}_u^l &= \frac{z(t) - q^n - w_u + \max\{-q^l + \alpha\Omega\}}{\tilde{\lambda}},\end{aligned}\quad (\text{A5})$$

with w_s, w_u the reservation wage of skilled and unskilled workers respectively, and $\Omega = U_s - U_u$ the incentives to acquire skills.

The reservation wages are well defined in the limit, and are given by

$$\begin{aligned}w_s &= b + \Delta\beta\tilde{f}(\theta_s^n)\max\{\tilde{S}_s^n, 0\} + (1 - \Delta)\beta\tilde{f}(\theta_s^r)\max\{\tilde{S}_s^r, 0\}, \\ w_u &= b + \frac{\Delta}{1 + I}\beta[\pi\tilde{f}(\theta_s^n)\max\{\tilde{S}_u^n, 0\} + (1 - \pi)\tilde{f}(\theta_u^l)\max\{\tilde{S}_u^l, 0\}] \\ &\quad + \frac{1 - \Delta}{1 + I}\beta\tilde{f}(\theta_u^r)\max\{\tilde{S}_u^r, 0\} + \frac{I}{1 + I}\beta\tilde{f}(\theta_u^o)\max\{\tilde{S}_u^o, 0\} + \delta\Omega.\end{aligned}\quad (\text{A6})$$

Finally, the normalized surpluses and γ^n determine the equilibrium tightness as:

$$\kappa \geq \tilde{q}(\theta_j^k)(1 - \beta)\mathbb{E}_S[\max\{\tilde{S}, 0\}|k, j]. \quad (\text{A7})$$

These equations mimic those obtained for the steady state, with the difference that x (or more precisely γ^n) and I are moving in the background and shifting all the equilibrium variables. In particular, z, x, I, Ω implicitly determine all equilibrium objects in the model as the solution to the system of equations given by (A4, A5, A6, A7). I refer to the corresponding values of surpluses, tightness and reservation wages as the *instantaneous equilibrium*.

Properties of the instantaneous equilibrium.

I start by proving the existence of an instantaneous equilibrium.

PROPOSITION A2 *For a set of given values of $z, x, I, \Omega \geq 0$, and $q^l > \bar{q}$, for a positive threshold \bar{q} , there exists at least one instantaneous equilibrium.*

PROOF. For a fixed set of values $z, x, I, \Omega \geq 0$, I define a mapping T from θ_s^n, w_s, w_u as follows:

1. θ_s^n, w_s, w_u determine $\tilde{f}(\theta_s^n)$ and the surplus of all jobs $\max\{\tilde{S}_j^k, 0\}$.
2. The surpluses determine the finding rates at jobs other than novel ones: \tilde{f}_j^k .
3. The finding rates \tilde{f}_j^k and $\tilde{f}(\theta_s^n)$ determine the total finding rates of workers: \tilde{f}_s and \tilde{f}_u .

4. The finding rates determine the share of unskilled workers among those that are searching for novel jobs:

$$\begin{aligned}\gamma^n &= \frac{\Delta\gamma}{\Delta\gamma + \frac{\Delta\pi}{1+I}(1-\gamma)}, \\ (1-\gamma) &= (1-x) \frac{\tilde{\lambda} + \gamma\tilde{f}_s + (1-\gamma)\tilde{f}_u}{\tilde{\lambda} + \tilde{f}_u}.\end{aligned}\tag{A8}$$

5. $T_{\theta_s^n}(\theta_s^n, w_s, w_u)$ is defined as the implied job creation in novel jobs, given the expected quality of matches determined by γ^n and the surpluses $\max\{\tilde{S}_n^n, 0\} > \max\{\tilde{S}_u^n, 0\}$. That is:

$$\kappa = \tilde{q}(T_{\theta_s^n}(\theta_s^n, w_s, w_u)) \left[\gamma^n \frac{z - w_s}{\tilde{\lambda}} + (1 - \gamma^n) \frac{z - q^n - w_u}{\tilde{\lambda}} \right],\tag{A9}$$

6. The mapping for reservation wages is defined as the implied value of searching, which is given by:

$$\begin{aligned}T_{w_s}(\theta_s^n, w_s, w_u) &= b + \Delta\beta f(\theta_s^n) \frac{z - w_s}{\tilde{\lambda}} + (1 - \Delta)\beta M(z - w_s), \\ T_{w_u}(\theta_s^n, w_s, w_u) &= b + \frac{\Delta}{1+I} \pi \beta f(\theta_s^n) \frac{\max\{z - q^n - w_u, 0\}}{\tilde{\lambda}} + \frac{1 - \Delta + I}{1+I} \beta M(z - w_u) \\ &\quad + \frac{\Delta}{1+I} (1 - \pi) \beta M(z - q^n + \max\{-q^l + \alpha\Omega, 0\} - w_u) + \delta\Omega.\end{aligned}\tag{A10}$$

Here, for each job performed by worker $j \in \{s, u\}$ at task of type $k \in \{n, r, l, o\}$, I denote by $M(z_j^k - w_j)$ the expected surplus obtained in these jobs, which includes the probability of finding the job. This expected surplus is given by $\tilde{f}(\theta_j^k) \max\{\tilde{S}_j^k, 0\}$, which depends solely on $z_j^k - w_j$.

An instantaneous equilibrium corresponds to a fixed point of this map: $T_{w_s}(\theta_s^n, w_s, w_u) = w_s$, $T_{w_u}(\theta_s^n, w_s, w_u) = w_u$, and $T_{\theta_s^n}(\theta_s^n, w_s, w_u) = \theta_s^n$. I now prove such a fixed point exists. The mapping T is continuous. Since surpluses are bounded and non-negative, it maps the compact set $[0, \bar{\theta}] \times [b, M(z) + \delta\Omega]^2$ into itself. Thus, Brouwer's fixed-point theorem implies that there exists a fixed point of this mapping, as wanted ■

I now provide conditions for the unicity of an equilibrium. To simplify my notation, let Φ be the vector of current values of z, x, I, Ω . I also assume $\delta \approx 0$ as in the main text. I present an equivalent construction of the equilibrium mapping that allows me to prove the unicity of a fixed point under certain conditions. Taking $w_s, w_u; \Phi$ as given, I define the tightness in novel jobs $\theta_s^n(w_s, w_u; \Phi)$ implicitly as the interception of equation (A8)—which determines the

average quality of the matching pool in novel jobs—and the equation (A9)—which determines the creation of novel jobs for a given expected quality and surpluses.

The following lemma provides a condition which I use to prove that $\theta_s^n(w_s, w_u; \Phi)$ is uniquely defined.

LEMMA A2 *For all values of Φ, w_s, w_u , such that $w_s \geq w_u$ and $z - w_s - q^n > 0$, we have that:*

$$\frac{1-\gamma}{\gamma} \frac{\pi}{1+I} < \frac{1-x}{x}.$$

PROOF. Because $w_s \geq w_u$ We have that $\tilde{f}_u^r = \tilde{f}_u^o > \tilde{f}_s^r > \tilde{f}_s^n$. This implies that $\tilde{f}_u > \frac{\pi}{1+I} \tilde{f}_s$, in the worst case in which $\tilde{f}_u^l = 0$. That is, unskilled workers can guarantee to find jobs at least at the rate at which they mimic skilled workers.

Therefore:

$$\begin{aligned} (1-\gamma) &= (1-x) \frac{\tilde{\lambda} + \gamma \tilde{f}_s + (1-\gamma) \tilde{f}_u}{\tilde{\lambda} + \tilde{f}_u} < (1-x) \frac{\tilde{\lambda} + \gamma \tilde{f}_s + (1-\gamma) \frac{\pi}{1+I} \tilde{f}_s}{\tilde{\lambda} + \frac{\pi}{1+I} \tilde{f}_s} \\ &< (1-x) \frac{\gamma \tilde{f}_s + (1-\gamma) \frac{\pi}{1+I} \tilde{f}_s}{\frac{\pi}{1+I} \tilde{f}_s} \\ &= (1-x) \frac{\gamma + (1-\gamma) \frac{\pi}{1+I}}{\frac{\pi}{1+I}}. \end{aligned}$$

The first inequality follows from the fact that $\tilde{f}_u > \frac{\pi}{1+I} \tilde{f}_s$. The second one follows by noting that $\gamma \tilde{f}_s + (1-\gamma) \frac{\pi}{1+I} \tilde{f}_s > \frac{\pi}{1+I} \tilde{f}_s$. The last line follows after canceling the term \tilde{f}_s in the numerator and denominator.

Rearranging the last expression yields $\frac{1-\gamma}{\gamma} \frac{\pi}{1+I} < \frac{1-x}{x}$, as wanted ■

Using this lemma, I now prove that $\theta_s^n(w_s, w_u; \Phi)$ is uniquely defined.

LEMMA A3 *For $w_s \geq w_u$, the tightness $\theta_s^n(w_s, w_u; \Phi)$ is unique. Moreover, $\theta_s^n(w_s, w_u; \Phi)$ and $\tilde{f}(\theta_s^n(w_s, w_u; \Phi))$ increase with x, I, Ω .*

PROOF. Equation (A9) defines an increasing locus between θ_s^n and γ^n in the (γ^n, θ_s^n) space, which I refer to as the job-creation locus. This follows from Lemma A1, which implies that $\frac{z-w_s}{\lambda} > \frac{z-q^n-w_u}{\lambda}$.

Equations (A8) determine a decreasing locus between γ^n and θ_s^n —which reflects the intuitive fact that when tightness in novel jobs increases, skilled workers find jobs faster and there are fewer skilled workers among the unemployed searching for novel jobs. I refer to this curve as the quality locus.

To show that this locus is decreasing, is equivalent to the claim that an increase in \tilde{f}_s^n —the finding rate of workers in novel jobs—reduces γ for fixed values of w_s, w_u, z, x, I, U . If $z - w_s - q^n < 0$, the increase in \tilde{f}_s^n raises f_s and does not affect f_u . Therefore, γ and γ^n

increase in the last two equations of (A8) as wanted. If $z - w_s - q^n > 0$, lemma A2 applies (recall that $w_s \geq w_u$). The increase in \tilde{f}_s^n raises f_s by Δ and f_u by $\frac{\Delta\pi}{1+I}$. Therefore, in the last two equations of (A8), we have that γ falls by $d\gamma = -\left(\frac{1-x}{x} - \frac{1-\gamma}{\gamma} \frac{\pi}{1+I}\right) \Delta df_s^n < 0$ and γ^n falls as well, which proves these equations describe a downward sloping locus, as depicted in Figure A1.

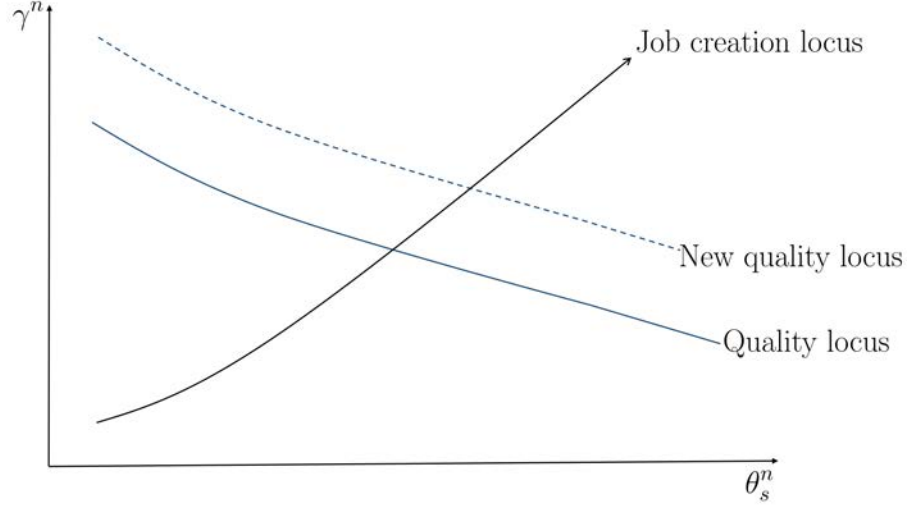


FIGURE A1: Quality locus (equation A8) and job-creation locus (equation A9).

To finalize, notice that for $\gamma^n = 0$ we have $f_s^n \geq 0$ in the job-creation locus. The quality locus crosses $f_s^n = 0$ at $\gamma^n \in (0, 1)$. Therefore, at $f_s^n = 0$, the job-creation locus is above the quality locus. As $f_s^n \rightarrow \infty$, the quality locus converges to $\gamma^n = 0$. Thus, both loci cross at a unique point that determines unique values for $\theta_s^n(w_s, w_u; \Phi)$ and $\gamma^n(w_s, w_u; \Phi)$.

Moreover, an increase in x , Ω and I shift the quality locus upwards, which implies that both $\gamma^n(w_s, w_u; \Phi)$ and $\theta_s^n(w_s, w_u; \Phi)$ increase in x , Ω and I for fixed values of w_s, w_u . The same comparative statics applies for $\tilde{f}(\theta_s^n(w_s, w_u; \Phi))$ ■

I denote the resulting finding rate in novel jobs as $\tilde{f}_s^n(w_s, w_u; \Phi) = \tilde{f}(\theta_s^n(w_s, w_u; \Phi))$, which increases in x, I, Ω . The effect of x, I, Ω through Φ capture the indirect effect of these variables through changes of the composition of workers that are searching for jobs—shifts in the quality locus. In my model, these indirect effects correspond to the extent to which different variables exacerbate or attenuate the skill mismatch.

The next proposition provides conditions for the uniqueness of the instantaneous equilibrium as a function of (z, x, I, Ω) .

PROPOSITION A3 *There exists a positive threshold \bar{q} such that, if $q^l > \bar{q}$ and $\beta\Delta \frac{\partial f_s^n(w_s, w_u; \Phi)}{\partial w_s}$ is uniformly bounded above by 1, the instantaneous equilibrium is unique.*

PROOF. I set \bar{q} as the threshold for which $q^l \geq \alpha\Omega - q^n + w_s - w_u$ if $q^l > \bar{q}$ (this threshold exists

since $w_s - w_u$ is bounded across all possible instantaneous equilibria). Thus, the condition $q^l > \bar{q}$ guarantees $S_u^l \leq S_s^n$, and therefore $w_s \geq w_u$ in any instantaneous equilibria.

Therefore, to find an instantaneous equilibrium, I may restrict to pairs (w_s, w_u) such that $w_s \geq w_u$. For any such pair, the function $\tilde{f}_s^n(w_s, w_u; \Phi)$ is well defined. Thus, the instantaneous equilibrium is fully determined by a pair of reservation wages $w_s \geq w_u$, which solve the fixed point problem:

$$\begin{aligned} T_s(w_s, w_u) &= b + \Delta \beta f_s^n(w_s, w_u; \Phi) \frac{z - w_s}{\tilde{\lambda}} + (1 - \Delta) \beta M(z - w_s), \\ T_u(w_s, w_u) &= b + \frac{\Delta}{1 + I} \pi \beta f_s^n(w_s, w_u; \Phi) \frac{\max\{z - q^n - w_u, 0\}}{\tilde{\lambda}} + \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u) \\ &\quad + \frac{\Delta}{1 + I} (1 - \pi) \beta M(z - q^n + \max\{-q^l + \alpha \Omega, 0\} - w_u), \end{aligned} \quad (\text{A11})$$

with $T_s(w_s, w_u)$ and $T_u(w_s, w_u)$ a continuous mapping which has at least one fixed point, and such that all fixed points satisfy $w_s \geq w_u$.

I now prove that a solution to the previous system is unique when $\beta \Delta \frac{\partial f_s^n(w_s, w_u; \Phi)}{\partial w_s}$ is uniformly bounded above by 1.

The function $f_s^n(w_s, w_u; \Phi)$ declines in w_u for two reasons: first, w_u reduces the surplus \tilde{S}_u^n . Second, w_u reduces f_u , which increases γ^n (see equation 9). Instead, $f_s^n(w_s, w_u; \Phi)$ may increase or decrease in w_s depending on how much do changes in w_s shift the quality and job-creation locus. Given my assumption on $\beta \Delta \frac{\partial f_s^n(w_s, w_u; \Phi)}{\partial w_s}$, the curve for w_s in equation (A11) describes a decreasing locus on the (w_s, w_u) space, as shown in Figure A2, while the curve for w_u describes another locus on the (w_s, w_u) space (which may be decreasing or increasing).

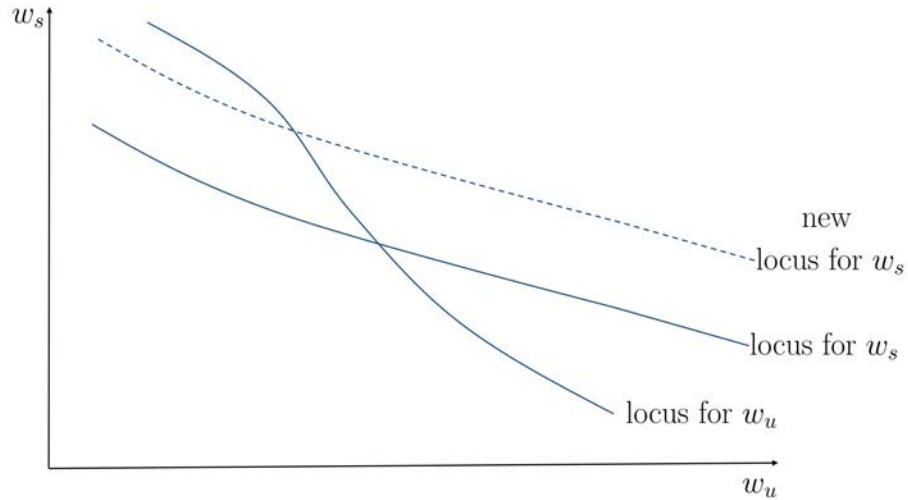


FIGURE A2: Loci for the fixed points of the map in equation (A3).

Suppose there are two equilibria (w_s^1, w_u^1) and (w_s^2, w_u^2) , with $w_s^1 < w_s^2$ without loss of generality. Since the locus determined by the equation for w_s is decreasing, we must have $w_u^1 > w_u^2$.

First, assume that $z > q^n + w_u^1$, so that all matches are formed in both equilibria. We have that:

$$\begin{aligned} \frac{w_s^i - (1 - \Delta)\beta M(z - w_s^i)}{\frac{z - w_s^i}{\lambda}} &= f_s^n(w_s^i, w_u^i; \Phi) \\ &= \frac{w_u^i - \frac{\Delta(1-\pi)}{1+I}\beta M(z - q^n + \max\{-q^l + \alpha\Omega, 0\} - w_u^i) - \frac{1-\Delta+I}{1+I}\beta M(z - w_u^i) - \delta\Omega}{\frac{\Delta}{1+I}\pi\beta\frac{z - q^n - w_u^i}{\lambda}}, \end{aligned} \quad (\text{A12})$$

with the left-hand side and the right-hand side, two increasing functions of w_s^i and w_u^i , respectively. Since $w_s^1 < w_s^2$, but $w_u^1 > w_u^2$, both equalities cannot hold simultaneously for $i = 1, 2$. This contradiction shows that there is a unique equilibrium.

Now, suppose that $z < q^n + w_u^1$. This implies that

$$\begin{aligned} w_u^1 &= b + \frac{\Delta}{1+I}(1 - \pi)\beta M(z - q^n + \max\{-q^l + \alpha\Omega - w_u^1, 0\}) + \frac{1 - \Delta + I}{1+I}\beta M(z - w_u^1) + \delta\Omega \\ &< b + \frac{\Delta}{1+I}(1 - \pi)\beta M(z - q^n + \max\{-q^l + \alpha\Omega - w_u^2, 0\}) + \frac{1 - \Delta + I}{1+I}\beta M(z - w_u^2) + \delta\Omega \\ &< b + \frac{\Delta}{1+I}\pi\beta f_s^n(w_s^2, w_u^2; \Phi) \frac{\max\{z - q^n - w_u^2, 0\}}{\tilde{\lambda}} \\ &\quad + \frac{\Delta}{1+I}(1 - \pi)\beta M(z - q^n + \max\{-q^l + \alpha\Omega - w_u^2, 0\}) + \frac{1 - \Delta + I}{1+I}\beta M(z - w_u^2) + \delta\Omega \\ &= w_u^2. \end{aligned}$$

This contradicts the fact that $w_u^1 > w_u^2$ and proves that there is only one equilibrium. ■

Remark: The bound on $\beta\Delta\frac{\partial f_s^n(w_s, w_u; z, x, I, \Omega)}{\partial w_s} < 1$ is akin to assuming that complementarities are weak. One can guarantee it in a number of ways. In particular, there are thresholds such that for $\beta < \bar{\beta}$, or for $\pi < \bar{\pi}$, the condition holds. Numerically, this condition is not too demanding, and for the parameters used in the calibration of my model the condition holds when $\beta = 0.5$ for all values of π . In the following propositions I provide a sharper characterization for \bar{q} , which is the one I used in the main text.

A1.4 Comparative statics for the instantaneous equilibrium.

I now provide comparative statics for the behavior of the instantaneous equilibrium. I concentrate in the case in which $q^l > \bar{q}$, and $\beta\Delta\frac{\partial f_s^n(w_s, w_u; \Phi)}{\partial w_s}$ is uniformly bounded above by 1, so that the equilibrium is unique.

Let $w_s(\Phi, z, x, I, \Omega)$ and $w_u(\Phi, z, x, I, \Omega)$ be the reservation wages in the instantaneous equilibrium which solve the fixed point problem given by equation (A11). Here, I introduce the argument Φ to separate the effect of all variables through $f_s^n(w_s^i, w_u^i; \Phi)$ —the effects through the quality of matches and the job creation externality—from the direct effects on the surpluses and matching rates. Likewise, denote by $f_s(\Phi, z, x, I, \Omega)$ and $f_u(\Phi, z, x, I, \Omega)$ the finding rates for both workers.

To save on notation I assume that the unique equilibrium is such that all surpluses are positive and all matches are formed, as is the case in my numerical exercise. All the results generalize to the case in which some matches are not formed with small modifications of the arguments that I present below.

PROPOSITION A4 *Suppose $q^l > \bar{q}$ and $\beta \Delta \frac{\partial f_s^n(w_s, w_u; \Phi)}{\partial w_s}$ is uniformly bounded above by 1, so that the equilibrium is unique. There exist thresholds $\bar{\pi}$ and $\bar{\beta}$ such that the instantaneous equilibrium satisfies the following properties:*

1. *Consider a change in Φ from Φ_1 to Φ_2 such that $f_s^n(w_s^i, w_u^i; \Phi_2) > f_s^n(w_s^i, w_u^i; \Phi_1)$. Then, w_s, w_u, f_s, f_u and $w_s - w_u$ increase. As a corollary, it follows that the external effect of x, I, Ω through γ^n is to increase w_s, w_u, f_s, f_u and $w_s - w_u$.*
2. *w_u increases in I and Ω . Moreover, if $\pi < \bar{\pi}$, we have that $w_u - w_s$ increases in I and Ω .*
3. *If $\beta < \bar{\beta}$ and $\pi < \bar{\pi}$ we have that both finding rates f_s, f_u increase in I and Ω . Moreover, γ^n, f_s^n and w_s also increase in I and Ω .*

PROOF OF NUMERAL 1: Notice that the increase in $f_s^n(w_s, w_u; \Phi)$ shifts the loci for $T_s(w_s, w_u) = w_s$ and $T_u(w_s, w_u) = w_u$ upwards and to the right in Figure A2. Since the locus for $T_s(w_s, w_u) = w_s$ is a decreasing curve, the new equilibrium must involve an increase in w_s or an increase in w_u , or both. However, equation (A12) implies that both reservation wages must move in the same direction, which implies that both w_s and w_u increase, and so does $f_s^n(w_s, w_u; \Phi)$.

In addition, since the left hand side of equation (A12) is less steep than the right hand side—which reflects the different frequencies with which workers match with novel jobs and the wage they obtain— w_s increases more than w_u , as wanted.

I now prove that both finding rates increase. As claimed above, $f_s^n(w_s, w_u; \Phi)$ increases. Moreover, due to the increase in reservation wages, the surplus obtained by skilled workers, which is given by $\tilde{S}_s^n = \tilde{S}_s^r = \frac{z - w_s}{\lambda}$, decreases.

Suppose by way of contradiction that f_s decreases. If this were the case and since $w_s = b + \beta f_s \frac{z - w_s}{\lambda}$, w_s would decline, which contradicts the fact that w_s increases. This contradiction implies f_s increases.

For unskilled workers we have that the surpluses $\tilde{S}_u^n = \frac{z - w_u - q^n}{\lambda}$, $\tilde{S}_u^l = \frac{z - w_u - q^n + \max\{-q^l + \alpha\Omega\}}{\lambda}$, and $\tilde{S}_u^r = \tilde{S}_u^o = \frac{z - w_u}{\lambda}$ decrease due to the increase in the reservation wage, w_u .

Let w_u increase from w_u^1 to w_u^2 , and denote by $\tilde{f}_j^k(\Phi)$ the resulting finding rates. We have:

$$\begin{aligned}
w_u^2 &= \frac{\Delta}{1+I} \beta \left[\pi \tilde{f}_s^n(\Phi_2) \frac{z - q^n - w_u^2}{\tilde{\lambda}} + (1 - \pi) \tilde{f}_u^l(\Phi_2) \frac{z - q^n - w_u^2 + \max\{-q^l + \alpha\Omega\}}{\tilde{\lambda}} \right] \\
&\quad + \frac{1 - \Delta + I}{1 + I} \beta \tilde{f}_u^r(\Phi_2) \frac{z - w_u^2}{\tilde{\lambda}} \\
&> \frac{\Delta}{1+I} \beta \left[\pi \tilde{f}_s^n(\Phi_1) \frac{z - q^n - w_u^1}{\tilde{\lambda}} + (1 - \pi) \tilde{f}_u^l(\Phi_1) \frac{z - q^n - w_u^1 + \max\{-q^l + \alpha\Omega\}}{\tilde{\lambda}} \right] \\
&\quad + \frac{1 - \Delta + I}{1 + I} \beta \tilde{f}_u^r(\Phi_1) \frac{z - w_u^1}{\tilde{\lambda}} \\
&> \frac{\Delta}{1+I} \beta \left[\pi \tilde{f}_s^n(\Phi_1) \frac{z - q^n - w_u^2}{\tilde{\lambda}} + (1 - \pi) \tilde{f}_u^l(\Phi_1) \frac{z - q^n - w_u^2 + \max\{-q^l + \alpha\Omega\}}{\tilde{\lambda}} \right] \\
&\quad + \frac{1 - \Delta + I}{1 + I} \beta \tilde{f}_u^r(\Phi_1) \frac{z - w_u^2}{\tilde{\lambda}}.
\end{aligned}$$

The first inequality uses the fact that $w_u^2 > w_u^1$, and the second uses the decline in surpluses as we move from Φ_1 to Φ_2 .

The last inequality implies that:

$$\begin{aligned}
(\tilde{f}_s^n(\Phi_2) - \tilde{f}_s^n(\Phi_1)) \frac{\Delta\pi}{1+I} &> (\tilde{f}_u^l(\Phi_1) - \tilde{f}_u^l(\Phi_2)) \frac{\Delta(1-\pi)}{1+I} \frac{\tilde{S}_u^l}{\tilde{S}_u^n} (\tilde{f}_u^r(\Phi_1) - \tilde{f}_u^r(\Phi_2)) \frac{1-\Delta+I}{1+I} \frac{\tilde{S}_u^r}{\tilde{S}_u^n} \\
&> (\tilde{f}_u^l(\Phi_1) - \tilde{f}_u^l(\Phi_2)) \frac{\Delta(1-\pi)}{1+I} + (\tilde{f}_u^r(\Phi_1) - \tilde{f}_u^r(\Phi_2)) \frac{1-\Delta+I}{1+I}.
\end{aligned}$$

The first line follows from rearranging the previous inequalities. The second line follows from the fact that \tilde{S}_u^n is smaller than the surplus obtained by unskilled workers in all other jobs.

This inequality implies that f_u increases after rearranging it ■

PROOF OF NUMERAL 2: Since $q^l > \bar{q}$, we have that $\tilde{S}_u^n \leq \tilde{S}_u^l < \tilde{S}_u^r = \tilde{S}_u^o$.

Therefore, for fixed values of w_s, w_u , an increase in I shifts the locus for $w_u = T_s(w_s, w_u)$ to the right. Through its effect on Φ , it also shifts the locus for $w_u = T_s(w_s, w_u)$ upwards. This implies that at least one among w_s, w_u increases.

Suppose w_s increases but w_u declines. The equation A12 implies that $f_s^n(w_s, w_u, \Phi)$ increases. Since w_u is given by

$$\begin{aligned}
w_u &= b + \frac{\Delta}{1+I} \pi \beta f_s^n \frac{\max\{z - q^n - w_u, 0\}}{\tilde{\lambda}} + \frac{1 - \Delta + I}{1 + I} \beta M(z - w_u) \\
&\quad + \frac{\Delta}{1 + I} (1 - \pi) \beta M(z - q^n + \max\{-q^l + \alpha\Omega, 0\} - w_u) + \delta\Omega,
\end{aligned}$$

and the increase in I shifts the right-hand side upwards (including f_s^n), w_u increases as well. This implies that w_u always increases.

Likewise, for fixed values of w_s, w_u , an increase in Ω shifts the locus for $w_u = T_s(w_s, w_u)$ to the right. Through its effect on Φ , it also shifts the locus for $w_u = T_s(w_s, w_u)$ upwards. This implies that at least one among w_s, w_u increases. Using the same argument as above, I

can discard a situation in which w_s increases but w_u declines, which implies that w_u always increases.

The threshold $\bar{\pi} > 0$ is defined by noting that, as $\pi \rightarrow 0$, the effect of both I and Ω on w_s converges to zero, while the effect of both I and Ω on w_u remains positive and bounded away from zero. Therefore, there exists a threshold $\bar{\pi}$ that guarantees that the direct effects of I and Ω on w_u dominate and $w_u - w_s$ increases in I, Ω ■

PROOF OF NUMERAL 3: The finding rate for unskilled workers is given by

$$f_u = \frac{\Delta}{1+I} \pi f_s^n + \frac{\Delta}{1+I} (1-\pi) f_u^l + \frac{1-\Delta+I}{1+I} f_u^r,$$

which already takes into account the fact that $f_u^r = f_u^o$.

The effect of a change in I on the finding rate can be written as:

$$df_u = \frac{\Delta}{(1+I)^2} \pi (f_u^r - f_s^n) dI + \frac{\Delta}{(1+I)^2} (1-\pi) (f_u^r - f_u^l) dI + \frac{\Delta}{1+I} \pi df_s^n + \mathcal{O}(\beta).$$

Here, the term $\mathcal{O}(\beta)$ stands for the change in w_s and w_u , which are of the same order of magnitude as β .

The condition $q^l > \bar{q}$ guarantees that $\tilde{S}_u^n \leq \tilde{S}_u^l < \tilde{S}_s^n \leq \tilde{S}_u^r = \tilde{S}_u^o$. This implies that $f_u^r > f_s^n$ and $f_u^r > f_u^l$.

These inequalities show that the term $\frac{\Delta}{(1+I)^2} \pi (f_u^r - f_s^n) dI + \frac{\Delta}{(1+I)^2} (1-\pi) (f_u^r - f_u^l) dI$ in the expression for df_u above is positive.

Turning to $d\gamma$, we have that

$$\begin{aligned} d\gamma &\propto \frac{1-\gamma}{\gamma} df_u - \frac{1-x}{x} df_s \\ &= \left(\frac{1-\gamma}{\gamma} \frac{\pi}{1+I} - \frac{1-x}{x} \right) \Delta df_s^n \\ &\quad + \frac{1-\gamma}{\gamma} \left(\frac{\Delta}{(1+I)^2} \pi (f_u^r - f_s^n) + \frac{\Delta}{(1+I)^2} (1-\pi) (f_u^r - f_u^l) \right) dI + \mathcal{O}(\beta). \end{aligned}$$

Thus, if $df_s^n < 0$, we would have that $d\gamma > 0$. But this implies $d\gamma^n > 0$ and we can write $df_s^n = A d\gamma^n + \mathcal{O}(\beta)$, and $df_s = A \Delta d\gamma^n + \mathcal{O}(\beta)$ with $A > 0$ given by the slope of the job-creation locus. If, on the other hand, we had $df_s^n > 0$, we would still have $df_s = \Delta df_s^n + \mathcal{O}(\beta)$.

In either case, the expressions for df_u , df_s^n and df_s imply that there exists a threshold $\bar{\beta} > 0$ such that, for $\beta < \bar{\beta}$, a rise in I increases f_u , f_s and f_s^n . Finally, since in this case the rise in I increases f_s^n , equation (A12) implies that w_s increases as well. The only way in which f_s^n may increase while both reservation wages, w_s and w_u , are larger, is if the composition γ_s^n improves.

Turning to the effect of an increase in Ω , we can write:

$$df_u = \frac{\Delta}{1+I} \pi df_s^n + \frac{\Delta}{1+I} (1-\pi) df_u^l + \mathcal{O}(\beta).$$

Here, $df_u^l > 0$ due to the increase in Ω , while $\mathcal{O}(\beta)$ takes into account the decline in f_u^r due to the increase in reservation wages.

Turning to $d\gamma$, we have that

$$\begin{aligned} d\gamma &\propto \frac{1-\gamma}{\gamma} df_u - \frac{1-x}{x} df_s \\ &= \left(\frac{1-\gamma}{\gamma} \frac{\pi}{1+I} - \frac{1-x}{x} \right) \Delta df_s^n + \frac{1-\gamma}{\gamma} \frac{\Delta}{1+I} (1-\pi) df_u^l + \mathcal{O}(\beta). \end{aligned}$$

Thus, if $df_s^n < 0$ we would have that $d\gamma > 0$. But this implies $d\gamma^n > 0$ and we can write $df_s^n = A d\gamma^n + \mathcal{O}(\beta)$, and $df_s = A \Delta d\gamma^n + \mathcal{O}(\beta)$ with $A > 0$ given by the slope of the job-creation locus. If, on the other hand, we had $df_s^n > 0$, we would still have $df_s = \Delta df_s^n + \mathcal{O}(\beta)$.

In either case, the expressions for df_u , df_s^n and df_s imply that there exists a threshold $\bar{\beta} > 0$ such that, for $\beta < \bar{\beta}$, a rise in Ω increases f_u , f_s and f_s^n . Finally, since in this case the rise in Ω increases f_s^n , equation (A12) implies that w_s increases as well. The only way in which f_s^n may increase while both reservation wages, w_s and w_u , are larger, is if the composition γ_s^n improves, as wanted ■

Remarks: The threshold $\bar{\pi}$ bounds the job creation externality. It guarantees that shocks affecting the surplus of unskilled workers (reductions in I or Ω) increase their incentives to become skilled. This regularity condition guarantees the saddle path stability of the system as I show below. In the main text, this condition is used to guarantee the uniqueness of the instantaneous equilibrium and this regularity condition that leads to the saddle path stability.

The condition $\beta < \bar{\beta}$ guarantees two things. First, it guarantees that increases in the surplus of one particular type of job do not reduce workers' finding rates. This counter-intuitive effect would result if, due to the increase in unskilled workers' reservation wage firms created less of other jobs and this resulted in a net decline in workers' finding rates. Second, the condition guarantees that, when unskilled workers are displaced from old jobs or few stepping-stone jobs are created, they reduce the expected surplus of job creation for firms that are posting novel jobs. This could fail to be the case if, due to the decline in their reservation wage, the abundant number of unskilled workers became more profitable matches in novel jobs than before. The condition $\beta < \bar{\beta}$ keeps unskilled workers' reservation wage from falling that much, so that the net effect of an inflow of displaced workers that are searching for novel jobs causes a reduction in job creation. I used this condition to guarantee that an improvement in the quality of potential matches, $d\gamma^n > 0$, is associated with the creation of more novel jobs.

The thresholds are not restrictive in my numerical exercise. In my calibration of the model, the instantaneous equilibrium is unique and $w_u - w_s$ is increasing in Ω and I for *any* value of $\pi \in [0, 1]$. Moreover, in my baseline calibration, one can have values for β as high as $\beta = 0.9$, for which I and Ω increase both workers finding rates and reservation wages. Thus,

these conditions are not demanding numerically, but are required conceptually.

A1.5 Proofs of the propositions 2-6.

The previous results allow me to prove the following generalization of Proposition 2 presented in the main text:

PROPOSITION A5 *Suppose we are in the limit case in which $a, \lambda \rightarrow \infty$. Moreover, assume $\pi < \bar{\pi}$ and $q^l > \bar{q}$. We have that:*

1. *The current values of z, x, I, Ω uniquely determine all equilibrium objects.*
2. *The equilibrium behavior of x, I, Ω boils down to the system of equations:*

$$\dot{x} = (1-x) \left[\alpha \frac{\frac{\Delta}{1+I}(1-\pi)\tilde{f}(\theta_u^l)}{\tilde{\lambda} + \tilde{f}_u} + \delta \right], \quad \dot{\Omega} = r\Omega + w_u - w_s \quad \dot{I} = -v(t)I,$$

coupled with an initial condition for $x(0), I(0)$.

3. *Near the unique steady state, the equilibrium is saddle-path stable.*
4. *The system is globally saddle path stable, with $\Omega(t)$ increasing monotonically to Ω^* .*
5. *The threshold \bar{q} is defined explicitly as the one that guarantees $\bar{q} = (r + \alpha)\Omega^* - q^n$.*

PROOF. Numerals 1 and 2 follow from the results presented above on the unicity and existence of the instantaneous equilibrium. The equilibrium behavior of x, I, Ω follows from rearranging equation (A4).

For numeral 3, we have that the behavior of the system can be approximated linearly around the steady state as (variables with an asterisk denote their steady state levels):

$$\begin{aligned} \dot{x} &= - \left[\alpha \frac{\Delta(1-\pi)f(\theta_u^{l*})}{\lambda + f_u^*} + \delta \right] (x - x^*) \\ \dot{I} &= -v(I - I^*) \\ \dot{\Omega} &= \left(\frac{\partial w_u}{\partial x} - \frac{\partial w_s}{\partial x} \right) (x - x^*) + \frac{\partial w_u}{\partial I} (I - I^*) + \left[r + \frac{\partial w_u}{\partial \Omega} \right] (\Omega - \Omega^*) \end{aligned}$$

The reason why there is no effect of I, Ω on x is because around the steady we have $x^* = 1$ and these effects are second order. Likewise, I has no effect on w_s near the steady state.

Locally, the system is recursive, with x and I converging monotonically to their steady state values at fixed rates, and Ω uniquely determined by forward integration over the resulting paths. To confirm this, notice that the linear system has two negative eigenvalues given by

$-\left[\alpha \frac{\Delta(1-\pi)f(\theta_u^l(\Omega^*))}{\lambda+f_u^*} + \delta\right]$ and $-v$, and a positive one given by $\left[r + \frac{\partial w_u}{\partial \Omega}\right]$. Since the system has two state variables and a forward looking variable, it is locally saddle path stable.

To extend this result outside a neighborhood of the steady state, notice that $I \rightarrow 0$ and $x \rightarrow 1$ always, which implies the global stability of the steady state.

I now turn to an analysis of the transitional dynamics. The equation for \dot{x} implies that $x(t)$ increases monotonically and converges to 1. The equation for \dot{I} implies that $I(t)$ decreases monotonically and converges to zero.

Consider the equilibrium when $z(t) = 1 \forall t$ and $I(t) = 0$. The equation for $\dot{\Omega}$ implies that as time goes by, x increases and $w_u - w_s$ declines (see Proposition A4). Also, as Ω increases, and since $\pi < \bar{\pi}$, we have that $w_u - w_s$ increases. Therefore, I can write the differential equation for Ω as $\dot{\Omega} = h(\Omega, t)$, with $h_\Omega > 0$ and $h_t < 0$.

The same holds when $z(t) = 1 \forall t$ but we also have $I(t) > 0$. In this case, and since $\pi < \bar{\pi}$ and $q^l > \bar{q}$, we have that the decline in $I(t)$ over time reduces $w_u - w_s$. Thus, in this case we also have that $\dot{\Omega} = h(\Omega, t)$, with $h_\Omega > 0$ and $h_t < 0$.

The steady state value for $\Omega(t)$ satisfies $\lim_{t \rightarrow \infty} h(\Omega^*, t) = 0$. Suppose that along the transition $\Omega(T) \geq \Omega^*$. Then $h(\Omega(T), T) > h(\Omega^*, t) = 0$, which implies $\dot{\Omega}(T) > 0$. Thus, for all $t > T$ we have $\Omega(t) > \Omega^*$ and $\dot{\Omega}(t) > 0$ so that $\Omega(t)$ is increasing, which contradicts the fact that in the unique steady state we always have $\lim_{t \rightarrow \infty} \Omega(t) = \Omega^*$. This contradiction implies that $\Omega(t) < \Omega^*$ along the transition.

Now, suppose that $h(\Omega(T), T) < 0$. Then $\Omega(t) < \Omega(T)$ for $t \in (T, T + \epsilon)$, for some $\epsilon > 0$. I claim that in this case $\Omega(t)$ declines for $t \geq T$. To prove it, suppose that it does not. Then there is a time $T' > T$ in which $h(\Omega(T'), T') = 0$ but $h(\Omega(t), t) < 0$ for $t \in [T, T')$. We have that $\Omega(t) > \Omega(T')$ for $t \in [T, T')$ by the election of T' . But this implies $0 > h(\Omega(t), t) > h(\Omega(T'), T') = 0$, a contradiction. This implies that $\Omega(t)$ declines for $t \geq T$, but then $\lim_{t \rightarrow \infty} \Omega(t) \neq \Omega^*$. This contradiction implies my initial supposition is false, and we have $h(\Omega(t), t) > 0$ for all t , which implies that $\Omega(t)$ increases monotonically until it reaches its steady state level.

I now prove the final numeral of the proposition. If $q^l > (r + \alpha)\Omega^* - q^n$, we have that $q^l > \alpha\Omega(t) + w_s - w_u - q^n$. This inequality uses the fact that $\Omega^* \geq \Omega(t)$ and $r\Omega \geq w_s - w_u$, since $\dot{\Omega} \geq 0$.

Therefore, $\bar{q} = (r + \alpha)\Omega^* - q^n$ is enough to guarantee the uniqueness of the equilibrium and the comparative statics results established above ■

Remark: In the main text I state all the propositions using the tighter condition $q^l > \bar{q}$, with $\bar{q} = (\alpha + r)\Omega^* - q^n$, which offers a sufficient characterization of \bar{q} that applies for all propositions proved here. This threshold is reasonable for several reasons. First, the condition $q^l > \bar{q}$ guarantees that $S_u^l < S_s^n$, $\Omega \geq 0$ and $w_s \geq w_u$. Thus, $q^l > \bar{q}$ is the right assumption required to obtain the intuitive feature that skilled workers have larger reservation wages

and produce a larger surplus in novel jobs than unskilled workers. I used these regularity conditions throughout the text when giving several intuitions, or describing the adjustment of the economy.

The condition is also satisfied in my numerical simulations. In the first numerical scenario, I have $\bar{q} = 0.34$, which is smaller than $q^l = 0.45$. In the second numerical scenario, I have $\bar{q} = 0.19$, which is smaller than $q^l = 0.3$.

Moreover, this condition is not restrictive. We have that $\bar{q} = (\alpha + r)\Omega^* - q^n = \alpha\Omega^* + w_s^* - w_u^* - q^n$. Lemma A1 implies that $w_s^* - w_u^* - q^n < 0$, which implies $\alpha\Omega^* > \bar{q}$. Thus, for $q^l \approx \bar{q}$ firms and workers face positive incentives to retrain.

Using the conditions $q^l > \bar{q}$, $\beta < \bar{\beta}$ and $\pi < \bar{\pi}$, I am now in a position to prove Propositions 3-5.

PROOF OF PROPOSITION 3: Since $q^n + q^l > (\alpha + r)\Omega$, we have that $q^n + q^l > \alpha\Omega + w_s - w_u$. Here, I used the fact that $\dot{\Omega} = r\Omega + w_u - w_s > 0$ along the adjustment (see Proposition A5). The inequality $q^n + q^l > \alpha\Omega + w_s - w_u$ implies that $\tilde{S}_s^n > \tilde{S}_u^l$.

Thus we have $\tilde{S}_s^n > \tilde{S}_u^n$ (by lemma A1) and $\tilde{S}_s^n > \tilde{S}_u^l$. These observations imply that the finding rate for unskilled workers can only be larger than that of skilled workers if $(1 - \Delta)(f_u^r - f_s^r)$ is large enough, or equivalently, if $w_s - w_u$ is large enough. The condition $\beta < \bar{\beta}$ guarantees this is not the case, and the increase in hiring in regular jobs does not fully compensate for the depressed finding rates of unskilled workers elsewhere.

The statement in the second numeral of the proposition follows from the observation that, along the transition we have that $\Omega(t)$ and $x(t)$ are increasing, as established in Proposition A4. Since f_s, f_u increase with both $\Omega(t)$ and $x(t)$, we have that $f_s(t)$ and $f_u(t)$ increase over time and approach their corresponding steady-state values.

The statement in the third numeral of the proposition follows from the observation that the stable arm for Ω is given by an increasing curve between Ω and x . Thus, a fall in $x(0)$ shifts the entire path for $x(t)$ and $\Omega(t)$ downwards. The comparative static results in Proposition A4 imply that f_s and f_u increase with both x and Ω . Therefore, a fall in $x(0)$ shifts down the entire path for $f_s(t)$ and $f_u(t)$ ■

PROOF OF PROPOSITION 4: The first part of the proposition follows as a corollary of Numeral 5 in Proposition A5. The second part follows from the fact that a low $x(0)$ shifts $\Omega(0)$ downwards along the stable arm of the system, which implies that it shifts down the entire equilibrium path for $\Omega(t)$ ■

Remark: Note that this result does not require any condition on β . This is because this proposition does not deal with the behavior of finding rates, but only of reservation wages.

I finalize this section with a proof of Proposition 5. As before, I assume we have $\pi < \bar{\pi}$, $\beta < \bar{\beta}$ and $q^l > \bar{q}$ so that the instantaneous equilibrium is unique and the comparative statics developed in Proposition A4 apply.

Before presenting the proof, I introduce some notation that I will use.

In order to separate the direct effects from those that operate through γ^n —that is, the job creation externality—, I define the functions $f_s^p(\gamma^n, z, I, \Omega)$ and $f_u^p(\gamma^n, z, I, \Omega)$, as the finding rates one would obtain for a fixed γ^n , which leave the quality of potential matches fixed.

The finding rates obtained once the change in γ^n is taken into account, are given by $f_s(z, x, I, \Omega)$ and $f_u(z, x, I, \Omega)$. These are defined by the unique solution to the system:

$$f_s = f_s^p(\gamma_n^p(f_s, f_n, x, I), z, I, \Omega) \quad f_u = f_u^p(\gamma_n^p(f_s, f_n, x, I), z, I, \Omega).$$

The function $\gamma_n^p(f_s, f_n, x, I)$ is defined implicitly by:

$$\begin{aligned} \gamma_n^p(f_s, f_n, x, I) &= \frac{\gamma(f_s, f_n, x)}{(1 - \gamma(f_s, f_n, x)) \frac{\pi}{1+I} + \gamma(f_s, f_n, x)}, \\ (1 - \gamma(f_s, f_n, x)) &= (1 - x) \frac{\lambda + \gamma(f_s, f_n, x)f_s + (1 - \gamma(f_s, f_n, x))f_u}{\lambda + f_u}. \end{aligned}$$

The comparative static results in Proposition A4, imply that both f_s and f_u increase with x . Moreover, we have that:

$$\begin{aligned} df_s &= \frac{\frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial x}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_u}} dx, \\ df_u &= \frac{\frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial x}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_u}} dx, \end{aligned}$$

Therefore, $\frac{\partial f_u^p}{\partial \gamma^n}, \frac{\partial f_s^p}{\partial \gamma^n} > 0$ —this relies on Numeral 1 of the comparative statistics, and $1 > \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_u}$ —this condition guarantees that compositional effects are not so strong as to have an improvement in match quality end up reducing finding rates. In fact, this condition follows from the fact that $\beta \Delta \frac{f_s^n(w_s, w_u; z, x, I, \Omega)}{\partial w_s} < 1$.

Using these functions, I am now in a position to prove Proposition 5.

PROOF OF PROPOSITION 5: We can write:

$$\begin{aligned} df_s &= \frac{\partial f_s^p}{\partial z} dz + \left(\frac{\partial \gamma_n^p}{\partial f_s} df_s + \frac{\partial \gamma_n^p}{\partial f_u} df_u \right) \frac{\partial f_s^p}{\partial \gamma^n}, \\ df_u &= \frac{\partial f_u^p}{\partial z} dz + \left(\frac{\partial \gamma_n^p}{\partial f_s} df_s + \frac{\partial \gamma_n^p}{\partial f_u} df_u \right) \frac{\partial f_u^p}{\partial \gamma^n}. \end{aligned}$$

This expression shows the effect of the decline in z can be decomposed in a direct effect—holding γ^n constant—, and an indirect effect through the decline in γ^n —the job creation externality.

The comparative static results presented above imply that $\frac{\partial f_s^p}{\partial \gamma^n}, \frac{\partial f_u^p}{\partial \gamma^n} > 0$. Moreover, holding γ^n constant we have $\frac{\partial f_u^p}{\partial z} > \frac{\partial f_s^p}{\partial z} > 0$, which is the standard effect of a change in productivity on finding rates, and is stronger on workers with lower productivity (see Shimer, 2005).

The solution to this system is given by:

$$\begin{aligned} df_s &= \frac{\partial f_s^p}{\partial z} dz + \frac{\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial z} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial z}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_u}} dz \\ df_u &= \frac{\partial f_u^p}{\partial z} dz + \frac{\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial z} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial z}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_u}} dz \end{aligned}$$

Here, the first term captures the direct effect of the recession, and the second term captures the way the recession exacerbates the skill mismatch by reducing the share of skilled workers who are searching for jobs.

This second term is positive (so that this mechanism reduces the finding rate for both workers) because:

$$\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial z} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial z} \propto \left(\frac{1 - \gamma}{\gamma} \frac{\partial f_u^p}{\partial z} - \frac{1 - x}{x} \frac{\partial f_s^p}{\partial z} \right) > 0. \quad (\text{A13})$$

The inequality follows from noting that $\frac{1-\gamma}{\gamma} > \frac{1-x}{x}$ (recall that this is the case because $f_s > f_u$), and $\frac{\partial f_u}{\partial z} > \frac{\partial f_s}{\partial z}$ because unskilled workers finding rate is more cyclical.

For small x , γ is small, which increases the cyclicity of the finding rate because f_u is more cyclical. In addition, a smaller x increases the response of γ^n to changes in the finding rates, which makes the term $d\gamma^n = \left(\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial z} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial z} \right) dz$ larger.

Now consider a change in I , given by dI . We can write

$$\begin{aligned} df_s &= \frac{\partial f_s^p}{\partial I} dI + \left(\frac{\partial \gamma_n^p}{\partial f_u} df_u + \frac{\partial \gamma_n^p}{\partial I} dI \right) \frac{\partial f_s^p}{\partial \gamma^n}, \\ df_u &= \frac{\partial f_u^p}{\partial I} dI + \left(\frac{\partial \gamma_n^p}{\partial f_u} df_u + \frac{\partial \gamma_n^p}{\partial I} dI \right) \frac{\partial f_u^p}{\partial \gamma^n}. \end{aligned}$$

The resulting change in finding rates is given by:

$$\begin{aligned} df_s &= \frac{\partial f_s^p}{\partial I} dI + \frac{\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial I} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial I}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_u}} dI \\ df_u &= \frac{\partial f_u^p}{\partial I} dI + \frac{\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial I} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial I}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_u}} dI. \end{aligned}$$

Moreover, we have that $d\gamma^n = \left(\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial I} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial I} \right) dI$. The comparative static results imply that γ^n increases with I , which implies that $\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial I} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial I} > 0$. Moreover, the condition $\beta < \bar{\beta}$ implies that, although $\frac{\partial f_s^p}{\partial I} < 0$ —since I reduces the profitability of matches with unskilled workers—this effect is dominated by $\frac{\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial I} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial I}}{1 - \frac{\partial f_s^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_s} - \frac{\partial f_u^p}{\partial \gamma^n} \frac{\partial \gamma_n^p}{\partial f_u}} > 0$.

As above, a when x is smaller the finding rate is more responsive both because there are more unskilled workers directly affected by I , and because $d\gamma^n = \left(\frac{\partial \gamma_n^p}{\partial f_s} \frac{\partial f_s^p}{\partial I} + \frac{\partial \gamma_n^p}{\partial f_u} \frac{\partial f_u^p}{\partial I} \right) dI$ is more responsive to changes in I .

To finalize the proof I need to specify how does Ω respond to changes in I and z . For a decline in z , Ω actually declines temporarily because, during the crisis, w_s falls more than w_z . Thus, this effect creates a force in the direction of increasing unemployment too. However, this effect is not quantitatively relevant since Ω is forward looking and the decline in productivity is only temporary.

In the case of a decline in I , Ω increases as a response. However, we have that $\Omega < \Omega^*$. Therefore, for large values of q^l , the response of job creation and the subsequent increase in x are not strong enough to compensate for the decline in I . Over time, x increases and the average finding rate and f_s return to their pretrend.

To prove the propagation can take as much time as wanted, consider the case in which $q^l > \alpha\Omega^*$, so that there is no training along equilibrium. In this case, the effect of I on f_s and f_u is permanent and x does not adjust to compensate for it. The effect of a decline in I only disappears asymptotically, when due to the exogenous acquisition of skills by unskilled workers, we have that $x = 1$ ■

A1.6 Proof of Proposition 6

PROOF OF THE CHARACTERIZATION OF THE CONSTRAINED EFFICIENT ALLOCATION: In the general case, the planner's problem is to maximize

$$\begin{aligned} \max \mathbb{W} = \int_0^\infty e^{-rt} & \left(u_s b + u_u b + \sum_{k,j} e_j^k(t) z_j^k - \kappa \theta_s^n \left[u_s \Delta + u_n \frac{\Delta}{1+I} \pi \right] - \kappa \theta_u^l u_u \frac{\Delta}{1+I} (1-\pi) \right. \\ & \left. - \kappa \theta_s^r u_s (1-\Delta) - \kappa \theta_u^r u_u \frac{1-\Delta}{1+I} - \kappa \theta_u^o u_u \frac{I}{1+I} \right) dt, \end{aligned}$$

subject to the behavior of the state variables described in equation (A1). Here, z_j^k is the product of a match, $z(t)$, net of training costs q^n and q^l .

The co-state for u_s , which I label Γ_s —and determines the social value of unemployment for skilled workers is given by

$$r\Gamma_s - \dot{\Gamma}_s = b + \Delta (f(\theta_s^n) SV_s^n - \kappa \theta_s^n) + (1-\Delta) (f(\theta_s^r) SV_s^r - \kappa \theta_s^r).$$

Here, SV_j^k denotes the value of a match $e_j^k(t)$, which is equal to the co-state for the state variable $e_j^k(t)$.

The co-state for u_u , which I label Γ_u —and determines the social value of unemployment for unskilled workers is given by

$$\begin{aligned} r\Gamma_u - \dot{\Gamma}_u = & b + \frac{\Delta}{1+I} \pi (f(\theta_u^n) SV_u^n - \kappa \theta_u^n) + \frac{\Delta}{1+I} (1-\pi) (f(\theta_u^l) SV_u^l - \kappa \theta_u^l) \\ & + \frac{(1-\Delta)}{1+I} (f(\theta_u^r) SV_u^r - \kappa \theta_u^r) + \frac{I}{1+I} (f(\theta_u^o) SV_u^o - \kappa \theta_u^o). \end{aligned}$$

The planer chooses tightness as to maximize the current value Hamiltonian. Thus, tightness is given by:

$$\kappa = f'(\theta_j^k) \mathbb{E}_{SV}[\max\{SV, 0\} | k, j].$$

This equation implies that

$$\kappa \theta_j^k = \theta_j^k f'(\theta_j^k) \mathbb{E}_{SV}[\max\{SV, 0\} | k, j] = (1 - \eta) f(\theta_j^k) \mathbb{E}_{SV}[\max\{SV, 0\} | k, j].$$

Replacing these terms in the reservation wages yields

$$r\Gamma_s - \dot{\Gamma}_s = b + \Delta f(\theta_s^n) (SV_s^n - (1 - \eta)[\gamma^n SV_s^n + (1 - \gamma^n) SV_u^n]) + \eta(1 - \Delta) f(\theta_s^r) SV_s^r,$$

and

$$\begin{aligned} r\Gamma_u - \dot{\Gamma}_u = & b + \frac{\Delta}{1 + I} \pi f(\theta_s^n) (SV_u^n - (1 - \eta)[\gamma^n SV_s^n + (1 - \gamma^n) SV_u^n]) + \frac{\Delta}{1 + I} (1 - \pi) f(\theta_u^l) SV_u^l \\ & + \frac{(1 - \Delta)}{1 + I} f(\theta_u^r) SV_u^r + \frac{I}{1 + I} f(\theta_u^o) SV_u^o. \end{aligned}$$

Equation 10 in the main text follows from these expressions after imposing the Hosios condition $\beta = \eta$ and noting that $\mu_j = r\Gamma_j - \dot{\Gamma}_j$. In addition, the values of employment SV_j^k satisfy the same Bellman equations derived for the decentralized economy, with Γ_j playing the role of U_j ■

PROOF OF THE COROLLARY TO PROPOSITION 6 IN THE MAIN TEXT: I prove that, in the limit case with large gross flows, subsidizing training at the margin increases welfare.

We have that along any given allocation:

$$\Gamma_s - \Gamma_u = \int_0^\infty e^{-rt} (\mu_s - \mu_u) dt > \int_0^\infty e^{-rt} (w_s - w_u) dt = \Omega.$$

Therefore, along the decentralized allocation $\widetilde{SV}_u^l > \widetilde{S}_u^l$.

I now prove that there is a path of non-negative subsidies for training that increase welfare in the decentralized allocation.

Suppose that firms creating stepping-stone jobs earn a surplus $S_u^l(t) + \sigma$, with $\sigma > 0$ for $t \in [0, T]$ denoting the subsidy.

For small values of σ , the subsidy increases tightness in stepping-stone jobs by $d_\theta(t) > 0$, and the effect on welfare is given by:

$$d\mathbb{W} = \int_0^T e^{-rt} \left[(1 - \eta) \widetilde{q}(\theta_u^l) \widetilde{SV}_u^l - \kappa \right] d_\theta(t) dt.$$

However, we have that along the decentralized allocation (and using the Hosios condition), $\kappa = (1 - \eta) \widetilde{q}(\theta_u^l) \widetilde{S}_u^l$. Therefore, the effect of a small subsidy to training is given by

$$d\mathbb{W} = \int_0^T e^{-rt} (1 - \eta) \widetilde{q}(\theta_u^l) \left[\widetilde{SV}_u^l - \widetilde{S}_u^l \right] d_\theta(t) dt > 0,$$

as wanted ■

A1.7 Allowing workers to direct their search efforts.

In this sub-section I describe an extension of my model in which I allow unemployed workers to direct their search efforts.

Each period unskilled workers draw an idiosyncratic shock $\varepsilon(i)$ determining their search efficiency when looking for jobs in task i . Moreover:

$$\varepsilon(i) \sim F(\phi, (1 + I)^{-1/\phi} \mu),$$

with $F(\phi, \mu)$ the Frechet distribution with shape $\phi > 1$ and scale μ . I normalize μ so that $\Gamma(1 - 1/\phi)\mu = 1$, and on average unskilled workers have one unit of search efficiency in the task they choose to search. After they obtain their draws, workers decide in which task they search for jobs. When their search efficiency is $\varepsilon(i)$ and they search for jobs in this task, their finding rate is $\varepsilon(i)f(\theta(i))$, with $\theta(i)$ the tightness at the task.

Workers search for jobs in the task that maximizes their expected utility from searching. With this distributional assumptions, the probability that the worker searches for a novel job is given by $\frac{\Delta}{1+I}\lambda_u^{n\phi-1}$, with

$$\lambda_u^n = \frac{\pi f(\theta_s^n)S_u^n + (1 - \pi)f(\theta_u^l)S_u^l}{\left(\frac{\Delta}{1+I} [\pi f(\theta_s^n)S_u^n + (1 - \pi)f(\theta_u^l)S_u^l]^\phi + \frac{1-\Delta}{1+I} [f(\theta_u^r)S_u^r]^\phi + \frac{I}{1+I} [f(\theta_u^o)S_u^o]^\phi\right)^{\frac{1}{\phi}}}.$$

The probability that the worker searches for a regular job is given by $\frac{1-\Delta}{1+I}\lambda_u^{r\phi-1}$, with

$$\lambda_u^r = \frac{f(\theta_u^r)S_u^r}{\left(\frac{\Delta}{1+I} [\pi f(\theta_s^n)S_u^n + (1 - \pi)f(\theta_u^l)S_u^l]^\phi + \frac{1-\Delta}{1+I} [f(\theta_u^r)S_u^r]^\phi + \frac{I}{1+I} [f(\theta_u^o)S_u^o]^\phi\right)^{\frac{1}{\phi}}}.$$

Finally, the probability that the worker searches for an old job is given by $\frac{I}{1+I}\lambda_u^{o\phi-1}$, with

$$\lambda_u^o = \frac{f(\theta_u^o)S_u^o}{\left(\frac{\Delta}{1+I} [\pi f(\theta_s^n)S_u^n + (1 - \pi)f(\theta_u^l)S_u^l]^\phi + \frac{1-\Delta}{1+I} [f(\theta_u^r)S_u^r]^\phi + \frac{I}{1+I} [f(\theta_u^o)S_u^o]^\phi\right)^{\frac{1}{\phi}}}.$$

We have that these probabilities collapse to $\frac{\Delta}{1+I}$, $\frac{1-\Delta}{1+I}$ and $\frac{I}{1+I}$ for $\phi \rightarrow 1$. Thus, the generalization presented here nests the model in the main text. The derivation for skilled workers is similar so I do not present it here.

For $\phi > 1$ workers direct their search efforts to tasks yielding a higher surplus, but only respond to differences in surpluses with an elasticity $\phi - 1$ —captured by the terms $\lambda_j^{k\phi-1}$ garbling the initial probabilities. Thus ϕ determines the extent to which workers may direct their search efforts. As $\phi \rightarrow \infty$, we converge to the case in which workers only search in their preferred task.

For intermediate values of ϕ , we have that as I declines, the probability that workers search for novel jobs, $\frac{\Delta}{1+I}\lambda_u^{n\phi-1}$ increases (both terms increase). Which generalizes to this environment the main force exacerbating the skill mismatch when $I(t)$ declines.

Likewise, if skilled workers had the chance to search for old jobs, we would have $\lambda_u^o > \lambda_u^n$ so long as $S_s^n > S_u^n$, which is always the case in equilibrium.

This exercise confirms that, if given the chance to direct their search efforts—albeit imperfectly—a reduction in old jobs $I(t)$ would still exacerbate the mismatch, since these jobs would hire more unskilled than skilled workers, and the unskilled would respond by moving to novel jobs.

A1.8 Behavior of wages when the number of open vacancies does not adjust immediately.

In this subsection I outline an extension of my model in which the number of open job vacancies does not adjust immediately, but does so gradually.

As mentioned in the main text, the gradual adjustment implies that, along the transition, the opportunity cost of a firm with an empty vacancy may depart from zero temporarily. This introduces an additional force that determines wages, which now depend on the change in both the worker's and the firm's outside options.

Formally, let $V_j^k(t)$ be the value of entering the market by opening a vacancy, v_j^k , at time t (to simplify the notation, I omit the task index, since equal tasks have an equal number of vacancies this is not needed). When $V_j^k(t) > 0$, a mass 1 of potential entrants are able to enter the market at a rate $\phi^{in} \in (0, \infty)$. When $V_j^k(t) < 0$, firms that hold an open vacancy are able to exit the market at a rate $\phi^{out} \in (0, \infty)$. When $V_j^k(t) = 0$ firms are indifferent between entering or exiting the market, and so the number of open vacancies does not change.

The model in the main text corresponds to the limit case in which $\phi^{in} = \phi^{out} = \infty$, which implies that the number of vacancies adjusts immediately and $V_j^k(t) = 0$ in equilibrium (if firms are entering the market). The assumption of a gradual adjustment of vacancies may be thought as a reduced form to capture irreversible investments (at least in the short run) made by firms to create jobs.

The value function V_j^k satisfies the Bellman equation:

$$rV_j^k - \dot{V}_j^k = -\kappa + \beta q(\theta_s^n) (\gamma^n \max\{S_s^n, 0\} + (1 - \gamma^n) \max\{S_u^n, 0\}) + \phi^{out} \max\{0, -V_j^k\}.$$

with $rV_j^k - \dot{V}_j^k = \rho_j^k$ the opportunity cost of the firm of engaging in the match. This equation shows that, when a firm enters the market, it must always pay the flow cost κ . I think of this assumption as a reduced form that incorporates the opportunity cost of the resources and

capital allocated to a particular job opening. The rate ϕ^{out} determines the speed at which the firm can redeploy these resources to other uses and close the vacancy.

The behavior of the number of open vacancies, v_j^k , satisfies:

$$\dot{v}_j^k = \phi^{in} \mathbb{1}\{V_j^k > 0\} - v_j^k \phi^{out} \mathbb{1}\{V_j^k < 0\}.$$

Nash bargaining implies that wages in different jobs are given by:

$$w_j^k = \beta z_j^k + (1 - \beta)(rU_j - \dot{U}_j) - \beta(rV_j^k - \dot{V}_j^k).$$

Unlike the case in which firms enter and exit the market immediately, now the wage also reflects the opportunity cost of the firm. Wages increase when $rV_j^k - \dot{V}_j^k$ is low because workers shield firms from having to stay with an open and unprofitable vacancy. Acemoglu (1997) discusses a similar effect in a model in which firms have to decide if they stay with their current match or they search for a new match.

Consider again the limit in which $a, \lambda \rightarrow \infty$. The following proposition shows that, when $x(0)$ declines, the wage of skilled workers at novel jobs increases temporarily.

PROPOSITION A6 *For any values of $\phi^{in}, \phi^{out} < \infty$, an unanticipated decline in γ^n —a worst skill mismatch—causes a temporary increase in the wage of skilled workers.*

PROOF. Since $\phi^{in}, \phi^{out} < \infty$, the number of vacancies for all jobs remain fixed at time 0.

This implies that skilled workers' job-finding rates and outside options remain fixed too, so that $w_s(0)$ does not change.

Lemma A1, implies that a decline in γ^n reduces $\rho_s^n(0)$ and $V_s^n(0)$. Importantly, the decline in $x(0)$ also reduces $\Omega(0)$, which creates a further decline in γ^n .

The equation for wages at time 0, implies that:

$$w_s^n(0) = \beta z(0) + (1 - \beta)w_s(0) - \beta\rho_s^n(0), w_s^r(0) = \beta z(0) + (1 - \beta)w_s(0) - \beta\rho_s^r(0).$$

Therefore, at time 0, $w_s^n(0)$ increases and $w_s^r(0)$ remains unchanged. Thus, the wage of skilled workers increases at time 0 and for a positive amount of time until vacancies adjust ■

The key implication of Proposition A6 is that, through its indirect effect on γ^n , a worst skill mismatch causes a temporary increase in the wage of skilled workers. We know from Proposition A4, that when vacancies adjust to keep $V_j^k = 0$ the worst skill mismatch reduces w_s and hence reduces the average wage of skilled workers. Thus, both results combined imply that, while vacancies adjust, we may have a temporary increase in the wage of skilled workers followed by a decline below its initial level.

This behavior of wages may explain the results found in Table 14 in the main text.

A1.9 Restructuring concentrates in recessions.

In the main text, I assumed that firms restructure their labor demand more during recessions. This assumption is a reduced-form way of capturing the idea that firms restructure during recessions.

In this subsection I discuss an extension of my model that endogenizes this feature.

Suppose that to produce an old task, firms need to purchase one unit of a capital good $m(i)$ produced by a monopolist for each $i \in [0, \Delta]$.

The monopolist produces the good at a marginal cost 0, but also faces a fixed cost of production C . Moreover, the monopolist prices the good at $p^m > 0$, which is exogenously determined by a fringe of competitive firms that could otherwise supply the capital good. For firms that post old jobs, p^m is a part of the recruiting cost κ , so that $\kappa > p^m$.

The monopolist is removed from the market and replaced by new technologies at a rate $v > 0$, which denotes the secular advancement of technology.

While the monopolist operates in the market, it may restructure its operation or liquidate its firm. Doing so allows the monopolist to loose the least value from its failed investment in the production of old tasks.

Restructuring (or liquidation) costs the monopolists R units of labor. This cost represents resources that are diverted away from production, and which are valued at an opportunity cost of $z(t)$. The assumption that the opportunity cost of restructuring is lower during recessions builds on the work of Hall (1991), and Aghion and Saint Paul (1998).

When the monopolist starts a restructuring process, it succeeds with Poisson probability $\bar{v} - v > 0$, in which case it pays the cost of restructuring and stops providing the capital good to firms producing the old task with labor.

Let V be the value of the monopoly. We have that

$$(r + v)V - \dot{V} = u(1 - \gamma)\theta_u^o \frac{1}{1 + I} p^m - C + (\bar{v} - v) \max\{-Rz(t) - V, 0\}.$$

Structural change lowers the value of the monopoly. Because of competition from technology and the fact that workers become skilled and stop searching for old jobs, we have that at some point $u(1 - \gamma)\theta_u^o \frac{1}{1 + I} p^m < C$ and the monopolist starts making negative profits.

Suppose that $z(t) = 1$ and $\frac{C}{r+v} > R$. Thus, there is a time T at which the firm decides to restructure. At this point, we have $V(T) = -R$, and $V(t) > -R$ for $t < T$.

In this case, a decline in productivity may prompt monopolists to restructure before time T . In particular, suppose that at time $T' < T$, productivity declines below $\frac{V(T')}{R} \in (0, 1)$, with $V(t)$ the value of the monopolist when the path for productivity is fixed and equal to $z(t) = 1 \forall t$. Due to the decline in productivity, the monopolist finds it profitable to liquidate the firm at time T' for two reasons. First, because the opportunity cost of assigning labor to

liquidate the firm is lower. Second, because recessions cause a large drop in vacancies, which make the monopoly less profitable. Thus, the value of retaining the monopoly falls.

Now, suppose that $z(t) = 1$ but $\frac{C}{r+v} > R$, so that firms would not liquidate along the adjustment and $V(t) \in (-R, 0)$ from some point onwards. In this case, a large productivity shock could also prompt restructuring efforts, which would not have happened otherwise.

Thus, recessions may cause an increase in the rate at which firms stop hiring labor for old jobs because firms front load the liquidation of old jobs to take advantage of the low opportunity cost to do so during recessions.

Remark: Instead of the fixed cost C , one could have that the monopolist pays a liquidation cost L if it is replaced by technology (this could be equal to $Rz(t)$ for the current value of $z(t)$ that determines productivity when the firm is replaced). Here, the difference in liquidation costs that the monopolist could save by restructuring during the recession plays the same role as C in the previous analysis. Moreover, a large L guarantees that the monopoly profits are negative along the whole transition because of the competition from technology, which is embedded in the liquidation cost.

When $L = Rz(t)$, the firm never restructures if $z(t)$ is constant and equal to 1, but it will do so if $z(t)$ declines temporarily below $\frac{v}{r+v}$.

A2 DATA APPENDIX

A2.1 Description of the data

Occupational groups: I use the 330 occupational groups proposed by Dorn (2009). These partition into consistently aggregated groups the occupations reported in the 1980, 1990 and 2000 Census and the American Community Survey (ACS). Military occupations are not included, and I exclude military personnel when using Census and ACS data. In recent waves of the ACS some small occupations have changed code or merged, but these changes do not cause problems when the data are aggregated.

For each occupational group I use the task-content measures developed by Acemoglu and Autor (2011) and Autor, David, and Murnane (2003). To measure training requirements, I use data from *O*NET* 8.0, which was released in 2005. For each job title the data include years of training and job-related experience required. I match job titles to SOC codes, and I use the available crosswalks to aggregate it to the 330 occupational groups in my data.

My division of jobs into broad occupational categories, including managerial, professional, routine-cognitive, routine manual and service jobs, follows Acemoglu and Autor (2011).

Commuting zones: I use 722 commuting zones that cover the entire continental U.S. but do not include Alaska and Hawai. David Dorn's crosswalks (available at his webpage

<http://www.ddorn.net/data.htm>) are used to aggregate Census and ACS geographic units at the commuting zone level. Recent waves of the American Community Survey use a new coding for Public Use Microdata Areas (PUMAs) to report geography. Using the available Census maps for the new PUMAs I do a geographic match to Counties in 1990. Then using David Dorn’s crosswalks I match Counties to commuting zones.

For the County Business Patterns data, I use the Acemoglu et al. (2014) codes to aggregate employment counts by consistently defined industries over time. This yields employment counts for each industry at the 4-digit SIC87 code and each County (using the 1990 delimitations) from 1988 to 2013. The Acemoglu et al. (2014) code also corrects for the fact that the CBP reports brackets for small industries. Using David Dorn’s crosswalk I aggregate these data at the commuting zone level.

Finally, I use the public-use data from the Longitudinal Employer-Household Dynamics. These data include figures on quarterly hirings and turnover for several (but not all) U.S. Counties. I aggregate these data to the commuting zone level and use them to construct Figure 3 as well as the complementary analysis that I present in the Appendix.

Skill groups: As explained in the text, I define skill groups by sex (2 categories), age (5 categories), educational attainment (5 categories), and region (4 regions), as reported in the Census and American Community Survey. This procedure yields a partition of the civilian workforce into 200 skill groups that I use in my analysis.

Measures of exposure to structural change: To define exposure to the decline of routine-cognitive jobs, I start by computing the occupations in the top tercile of routine-cognitive content. I borrow the index of routine-cognitive content from Acemoglu and Autor (2011), who construct it on the basis of $O * NET$ data about the type of tasks performed on the job.

Using the 2000 Census, I define RC_i as the share of workers who in each commuting zone worked outside manufacturing and were last employed in the top tercile of jobs that had the greatest routine-cognitive content.

Likewise, using the 2000 Census, I define GRC_g as the share of workers in each skill group who worked outside manufacturing and were last employed in the top tercile of jobs that had the greatest routine-cognitive content.

Geographic distribution of the commuting zone characteristics analyzed in the paper: For a detailed analysis of the geographic distribution of commuting zones that specialized in routine-cognitive jobs see Autor, Dorn and Hanson (2013b). Here I present maps for the main commuting zone characteristics used in my study. The required boundary files are available from Michael Stepner’s website <https://michaelstepner.com/>.

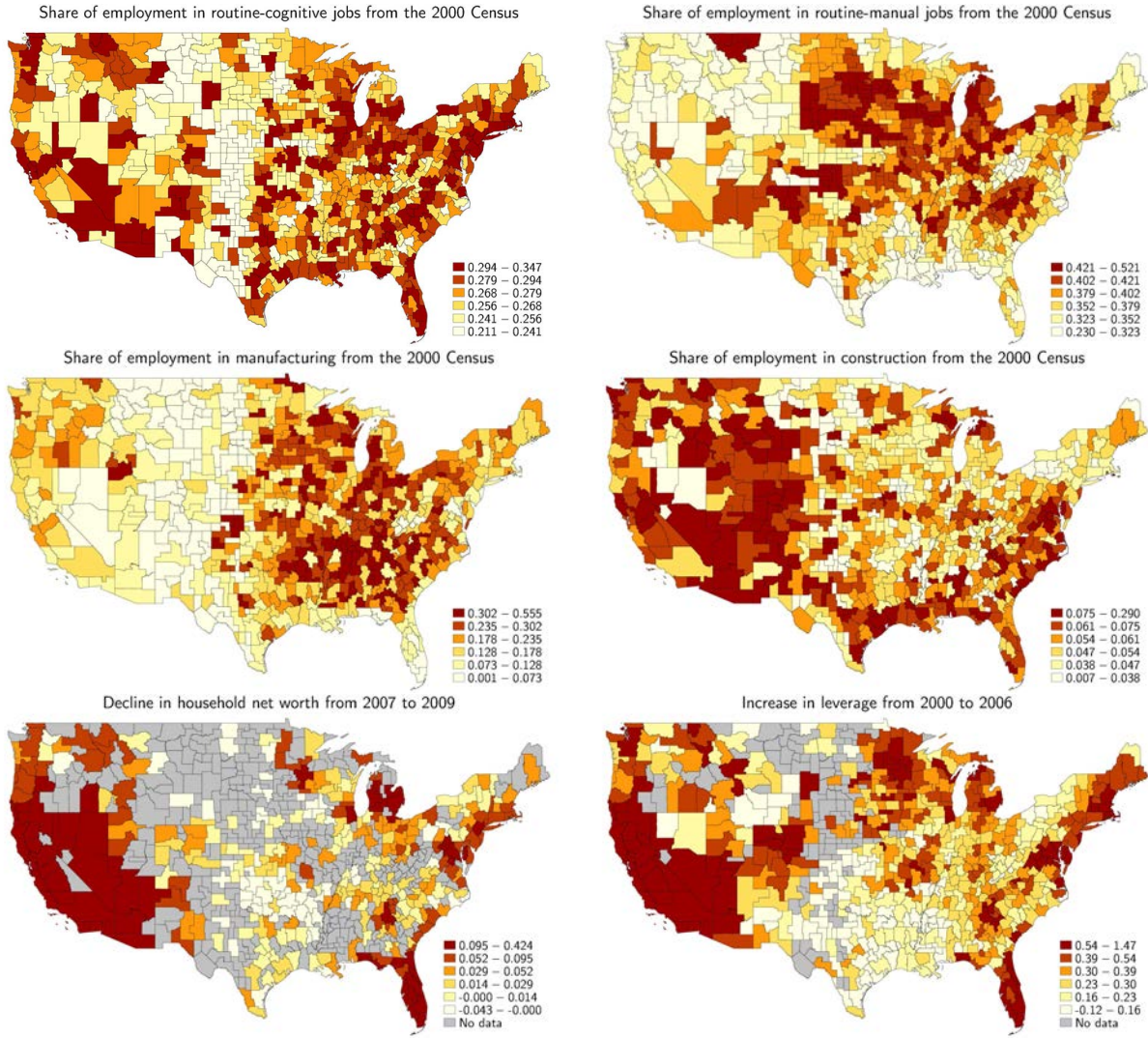


FIGURE A3: Geographical variation of the main commuting-zone characteristics used in my analysis.

A2.2 Routine-manual and professional jobs

I also construct measures of employment in routine-manual jobs in manufacturing (routine-manual jobs), which take into account the decline in manufacturing and production jobs precipitated by automation. This yields a measure of exposure to routine-manual jobs at the commuting-zone level RM_i , and a measure of specialization for skill groups in routine-manual jobs, GRM_g .

I control for these measures in my analysis in the main text, which distinguish my effects from the secular decline in production jobs in manufacturing. I do not emphasize the point estimates for these terms in the main text because the fact that manufacturing industries are more cyclical complicates the interpretation of these estimates (see Foote and Ryan, 2014).

In this appendix, I present estimates of equation (13) that include the direct and external effects of the decline of routine jobs in manufacturing during the Great Recession.

In addition, I include a measure of the availability of professional jobs in each commuting zone, which serves as a proxy for the number of skilled workers in each local economy. Because a large number of workers who specialize in professional jobs mitigates the skill mismatch, this variable should play the opposite role of the share of employment in routine-cognitive jobs.

TABLE A1: Direct and external effects of mismatch on employment, unemployment and wages.

<i>Dependent variable:</i>	Recession years			Recovery years		
	Change from 2007 to 2009-2010			Change from 2007 to 2011-2013		
	Employment (1)	Unemployment (2)	Wages (3)	Employment (4)	Unemployment (5)	Wages (6)
CZ's exposure to routine-cognitive jobs, RC_i	-0.123*** (0.030)	0.115*** (0.021)	-0.267*** (0.048)	-0.148*** (0.044)	0.113*** (0.029)	-0.433*** (0.075)
Skill group's specialization in routine-cognitive jobs, GRC_g	-0.089*** (0.022)	0.064*** (0.014)	0.032 (0.030)	-0.099*** (0.023)	0.027** (0.011)	0.056 (0.034)
CZ's exposure to routine-manual jobs, RM_i	-0.029 (0.019)	0.009 (0.017)	-0.028 (0.035)	-0.020 (0.022)	-0.009 (0.017)	-0.017 (0.048)
Skill group's specialization in routine-manual jobs, GRM_g	-0.127*** (0.026)	0.077*** (0.014)	-0.025 (0.031)	-0.127*** (0.027)	0.031** (0.014)	0.006 (0.039)
CZ's exposure to professional jobs	0.308*** (0.062)	-0.262*** (0.043)	0.252** (0.108)	0.203*** (0.065)	-0.142*** (0.035)	0.239 (0.170)
R squared	0.19	0.17	0.06	0.19	0.13	0.11
Observations	70534	70534	70232	106259	106259	105907

Notes: The table presents estimates of the differential change in labor market outcomes from 2007 onward among commuting zones more exposed to structural change (the external effect), together with the differential changes for workers in skill groups directly exposed to structural change (the direct effect). The dependent variable is indicated in top of each column, as well as the period for which I estimate the model. In all models, I allow the error term ε_{igt} to be correlated within States and over time, and within skill groups and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, I weight each commuting zone \times skill group cell by its size in 2000.

Table A1 presents my results. I find that the exposure to automation within manufacturing had no external effect on employment, but it had a negative direct effect on workers in skill groups that specialized in these jobs. This finding is consistent with the fact that workers displaced by automation in manufacturing reallocated mostly to service jobs. This produces no skill mismatches because these jobs do not require new skills or intensive training (see Autor and Dorn, 2013).

In line with the predictions of my model, I find that the commuting zones where the number of professional jobs was large before the recession experienced less unemployment during the recession and its recovery. During the Great Recession, the external effects of the skill mismatch were the largest in zones with many workers employed in routine-cognitive jobs, and few existing professionals.

A2.3 Results using the Longitudinal Employer-Household Dynamics data

My model predicts that low job-finding rates drive the increase in joblessness. To determine whether this is the case I examine data from the Longitudinal Employer-Household Dynamics, which reports total hirings and turnover at a quarterly level by industry and county. I aggregate the data by year at the commuting zone level and compute the annual hiring rate as total hirings over the amount of workers who had no job in that year.

TABLE A2: Hiring and separation rates among commuting zones affected by structural change, during and before the Great Recession.

	All industries			Non-manufacturing industries		
	Change from 2007 to 2010	Change from 2007 to 2013	Change from 2005 to 2007	Change from 2007 to 2010	Change from 2007 to 2013	Change from 2005 to 2007
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Panel A. Dependent variable: annual hiring rate from LEHD.</i>						
Share of routine jobs outside manufacturing	-1.525*** (0.448)	-1.170** (0.542)	-0.230 (0.266)	-1.472*** (0.443)	-1.124** (0.518)	-0.189 (0.246)
Share of routine jobs in manufacturing	0.170 (0.161)	0.145 (0.207)	0.099 (0.147)	0.128 (0.152)	0.128 (0.197)	0.099 (0.139)
R-squared	0.57	0.45	0.18	0.60	0.47	0.17
Observations	698	698	698	698	698	698
<i>Panel B. Dependent variable: annual turnover rate from LEHD.</i>						
Share of routine jobs outside manufacturing	-0.154** (0.060)	-0.217* (0.112)	-0.020 (0.042)	-0.354*** (0.107)	-0.294*** (0.088)	0.053 (0.046)
Share of routine jobs in manufacturing	0.090** (0.038)	0.101* (0.059)	0.037 (0.022)	0.137** (0.061)	0.162*** (0.049)	0.028 (0.029)
R-squared	0.57	0.54	0.20	0.18	0.19	0.07
Observations	698	669	698	633	611	645

Notes: The table presents estimates of the differential change in labor market outcomes from 2007 onward among commuting zones that were highly exposed to structural change. The dependent variable is the change in the annual hiring rate (Panel A), and the turnover rate (Panel B). The change in the dependent variable is computed over the years indicated on top of each column. Columns 1 to 3 use data for all industries, while Columns 4 to 6 use only data for non-manufacturing industries. When estimating this equation, I allow the error term ε_{it} to be correlated within States and over time, and I compute standard errors that are robust to this correlation structure and to heteroskedasticity. Finally, following a common practice in the literature, I weight commuting zones by the size of their workforce in 2000.

Table A2 presents my estimates of equation 12 when I use these outcomes. Columns 1 to 3 present estimates for all industries; while Columns 4 to 6 focus on hires and separations outside of manufacturing. I find that the increase and persistence in joblessness is driven by a significant decline in the hiring rate. The estimates in this table imply that a 10 percentage point increase in exposure to structural change is associated with a 14 percentage point reduction in the annual hiring rate during the onset of the Great Recession and its recovery (standard error=4.5 and 4.7, respectively). This corresponds to a 14% decline in the average finding rate in the data, and this in turn translates into a 1 percentage point increase

in unemployment—a figure that matches my previous estimates.³⁶ Moreover, Columns 4 to 6 show that this effect is driven entirely by a reduction of hires within non-manufacturing industries. Figure 3 illustrates the same result when the full quarterly data is analyzed. In Panel B, I also find that the annual turnover rate—defined by LEHD as the rate at which jobs begin and end—decreased in markets that during the recession were exposed to structural change.

³⁶The LEHD data yields an annual hiring rate of 1. This is considerably smaller than the finding rate reported by Shimer (2005). The difference arises because I compute the rate per worker without a job rather than per unemployed worker. Using the rate per unemployed worker yields similar but less precise estimates.